B03. **MATH 1700: Test #3 (Fall 2011)**

Solution; marking scheme

[17] 1. Consider the curves $y=-x^2+1$ and $y=0$. They bound a regions *R*.

[3] (a) Sketch *R*.

[7] (b) Set up, but do NOT evaluate the integral for volume of the solid obtained by rotating *R* around the vertical line $x=1$. Use the method of shells.

[7] (c) Set up, but do NOT evaluate the integral for volume of the solid obtained by rotating *R* around the vertical line $x=1$. Use the method of washers.

Solution.

(a) *R* is a convex parabola crossing the *x*-axis at -1 and 1 (sketch …).

(b) $\int_{-1}^{1} 2\pi(1-x)(1-x^2)\,dx$.

(c) $\int_{0}^{1}\left(\pi\left(1+\sqrt{1-y}\right)^2-\pi\left(1-\sqrt{1-y}\right)^2\right)dx$.

[both (b) and (c) yield 8/3.]

[7] 2. [4] (a) Find the inverse function of $y=\dfrac{2x+1}{x-1}$.

[3] (b) Differentiate $f(x)=\left(\cos^{-1}x\right)(\sin^{-1}x)$ with respect to *x*.

Solution.

(a) Solving for *x* in terms of *y*:

$y=\dfrac{2x+1}{x-1} \Leftrightarrow y(x-1)=(2x+1) \Leftrightarrow yx-2x=y+1 \Leftrightarrow x=\dfrac{y+1}{y-2}$; so the inverse is $y=\dfrac{x+1}{x-2}$.

(b) $f'(x)=\dfrac{-1}{\sqrt{1-x^2}}(\sin^{-1}x)+\dfrac{1}{\sqrt{1-x^2}}(\cos^{-1}x)$.