B02. **MATH 1700: Test #2 (Fall 2011)**

Solution; marking scheme

[8] 1. Write the definite integral $\int_a^b f(x)\,dx$ that corresponds to the following limit. (In both cases your answer should be in the form $\int_a^b f(x)\,dx$ with some specific $a, b,$ and $f(x)$; no need for justification.)

[4] (a) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{3-1}{n}\right)2011$

[4] (b) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3-1}{n}\sqrt{1+i\frac{2}{n}}$

Solution. (a) $\int_1^3 2011\,dx$ (b) $\int_1^3 \sqrt{x}\,dx$

[7] 2. Find the derivative: $\frac{d}{dx}\left(\int_{x^3}^{x^2}\cos t\,dt\right)$. Do not simplify after differentiating.

Solution.

$$\frac{d}{dx}\left(\int_{x^3}^{x^2}\cos t\,dt\right)=\frac{d}{dx}\left(\int_{x^3}^{0}\cos t\,dt\right)+\frac{d}{dx}\left(\int_{0}^{x^2}\cos t\,dt\right)=$$

$$=\frac{d}{dx}\left(-\int_{0}^{x^3}\cos t\,dt\right)+\frac{d}{dx}\left(\int_{0}^{x^2}\cos t\,dt\right)=-\left(\cos x^3\right)3x^2+\left(\cos x^2\right)2x$$

[10] 3. Evaluate the following integrals. Do not simplify.

[5] (a) $\int\frac{x}{x+1}\,dx$

[5] (b) $\int_1^2\frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx$

Solution. (a) Use $u = x+1$, so $du = dx$ and

$$\int\frac{x}{x+1}\,dx=\int\frac{u-1}{u}\,du=\int\left(1-\frac{1}{u}\right)du=u-\ln|u|+c=(x+1)-\ln|x+1|+c\,.$$

(b) Use $u=\sqrt{x}$, so that $du=\frac{1}{2\sqrt{x}}dx$:

$$\int_1^2\frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx=\int_{x=1}^{x=2}\cos u\,du=\sin u\Big|_{x=1}^{x=2}=\sin(\sqrt{x})\Big|_{x=1}^{x=2}=\sin(\sqrt{2})-\sin(\sqrt{1})\,.$$