B01. **MATH 1700: Test #1 (Fall 2011)**

Solution; marking scheme

[13] 1. Use L'Hospital rule to evaluate the following limits.

[5] (a) $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x}$

[8] (b) $\lim\limits_{x\to 0^+} x^x$

Solution.

(a) $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x} = (type\,{}^0\!/_0) = \lim\limits_{x\to 0} \dfrac{2x\cos x^2}{1} = 0$

(b) Write $y = x^x$. So $\ln y = \ln x^x = x\ln x$. We have

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = (type\,{}^{-\infty}\!/_{\infty}) = \lim_{x\to 0^+} \frac{{}^1\!/_x}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = 0 .$$

So $\lim\limits_{x\to 0^+} \ln y = 0$; hence $\ln\left(\lim\limits_{x\to 0^+} y\right) = 0$; hence $\lim\limits_{x\to 0^+} y = e^0 = 1$.

[5] 2. Sketch the following curve defined through parametric equations: $x = 3\cos 2t$, $y = -3\sin 2t$, $t \in (-\infty,\infty)$. Hint: eliminate the parameter to find the Cartesian equation of the curve.

Solution. $x^2 + y^2 = 9$, so this is the circle centered at the origin with radius 3. (Sketch)

[7] 3. Find the slope of the tangent line to the curve $r = 2\theta$ (given in polar coordinates) at the point when $\theta = \dfrac{\pi}{2}$. Simplify.

Solution. We have $x = r\cos\theta = 2\theta\cos\theta$, and $y = r\sin\theta = 2\theta\sin\theta$. So,

$\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \dfrac{2\sin\theta + 2\theta\cos\theta}{2\cos\theta - 2\theta\sin\theta}$. The slope is the value of this expression when $\theta = \dfrac{\pi}{2}$, which is $-\dfrac{2}{\pi}$.