Values

[12] 1. Evaluate.

(a) $\lim_{x\to 0}\frac{e^{3x}-3x-1}{x^2}$ $\left(\frac{0}{0}\right)$

$\overset{H}{=}\lim_{x\to 0}\frac{3e^{3x}-3}{2x}$ $\left(\frac{0}{0}\right)$

$\overset{H}{=}\lim_{x\to 0}\frac{9e^{3x}}{2}=\frac{9}{2}$

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(b) $\lim_{x\to\infty}(1+3x)^{\frac{2}{x}}$ (1^0)

Let $y=(1+3x)^{2/x}$

$\ln y=\frac{2}{x}\ln(1+3x)$

$\lim_{x\to\infty}(\ln y)=\lim_{x\to\infty}\frac{2\ln(1+3x)}{x}$ $\left(\frac{\infty}{\infty}\right)$

$\overset{H}{=}\lim_{x\to\infty}\frac{\frac{2}{1+3x}(3)}{1}=0$

$\therefore \lim_{x\to\infty}(1+3x)^{2/x}=e^0=1$

7

[5] 2. Find the value of $\frac{d^2y}{dx^2}$ when $t = 0$ for the curve defined by the parametric equations:

$$x = e^t \qquad y = \sin t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{e^t} = y' = \cos t e^{-t}$$

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{-\sin t e^{-t} - \cos t e^{-t}}{e^t}$$

at $t=0$

$$\frac{d^2y}{dx^2} = \frac{0 - 1e^{-0}}{e^0} = -1$$

[12] 3. Evaluate.

(a) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

Let $u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

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$$= \int \frac{\cos u}{u}(2u\,du)$$

$$= 2\int \cos u\,du = 2\sin u + c$$

$$= 2\sin\sqrt{x} + c$$

OR

DIRECTLY

(b) $\int_0^3 x^3\left(\sqrt{25 - x^2}\right)dx$

Let $u = \sqrt{25-x^2}$

$u^2 = 25 - x^2$

$2u\,du = -2x\,dx$

$-u\,du = x\,dx$

x	u
0	5
3	4

$$= \int_0^3 x^2\sqrt{25-x^2}\,(x\,dx)$$

$$= \int_5^4 (25-u^2)u(-u\,du)$$

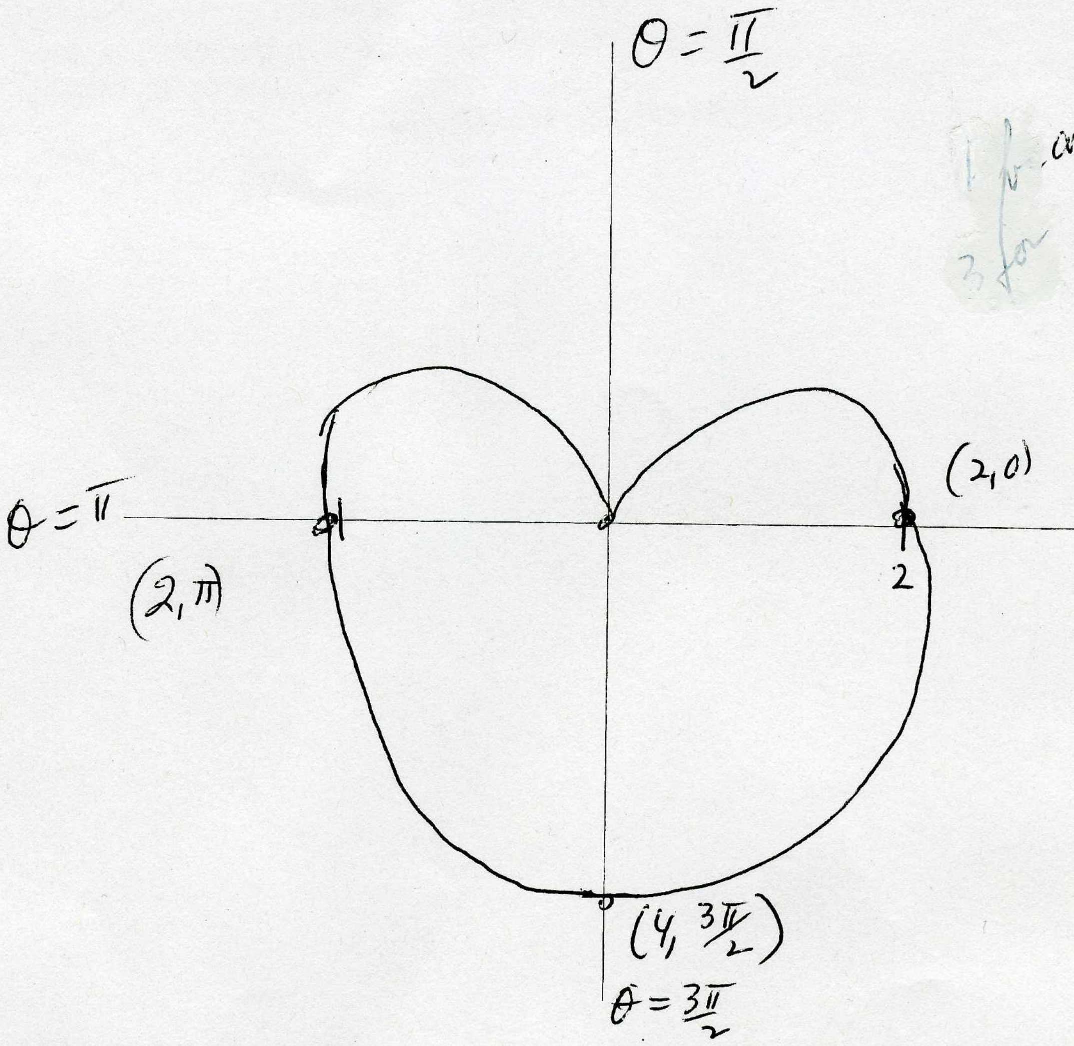
$$= \int_5^4 (u^4 - 25u^2)\,du$$

7

$$= \frac{u^5}{5} - \frac{25u^3}{3}\Big]_5^4$$

$$\ldots)^3 \quad (5^5 \ldots 25(5^3))$$

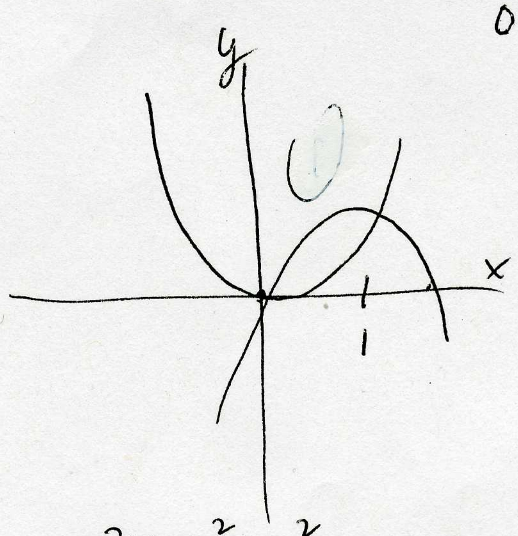
[4] 4. Sketch the curve with polar equation $r = 2 - 2\sin\theta$. Be sure to state the coordinates of all pertinent points.



1 for coordinates
3 for graph

[19] 5. (a) Set up, but <u>DO NOT EVALUATE</u>, the integral representing the area of the region bounded by the curves $y = 2x - x^2$ and $y = x^2$.

$0 = x(2 - x)$



$$\text{Area} = \int_0^1 \left[(2x - x^2) - x^2\right] dx$$

1 1 1

5

$2x - x^2 = x^2$

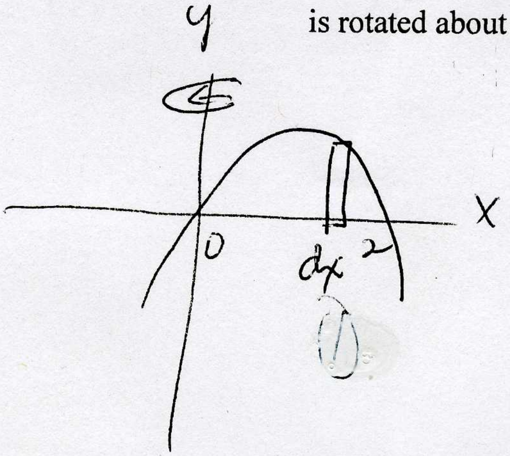
$0 = 2x^2 - 2x$

$0 = 2x(x - 1)$

5. (d) Set up, but <u>DO NOT EVALUATE</u>, the integral representing the volume of the solid formed when the region bounded by the curves

$$y = 2x - x^2 \text{ and } y = 0$$

is rotated about y-axis.



$0 = x(2-x)$ $\quad 2\pi r h$

$x = 0, 2$

$$V = \int_0^2 2\pi \cancel{y}\, x (2x - x^2)\, dx$$

1 1 1 1

5

[5] 6. Use the <u>definition</u> of a definite integral to express $\int_0^3 x^2 dx$ as a limit of a Riemann sum. <u>DO NOT EVALUATE THIS SUM</u>.

$\Delta x = \frac{3}{n}$

$$\int_0^3 x^2 dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{3i}{n}\right)^2 \left(\frac{3}{n}\right)$$

$x_i^* = \frac{3i}{n}$

using right endpoints

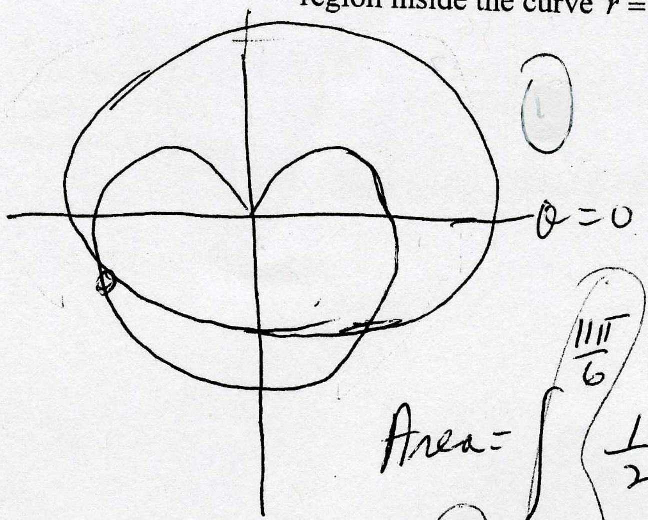
1 1 2 1

[3] 7. Find the derivative of the function $f(x) = \int_{x^3}^{2} \frac{\sin t}{t} dt$

$$f(x) = -\int_2^{x^3} \frac{\sin t}{t} dt$$

$$f'(x) = -\frac{\sin x^3}{x^3}(3x^2)$$

5. (b) Set up, but <u>DO NOT EVALUATE</u>, the integral representing the area of the region inside the curve $r = 2 - 2\sin\theta$ and outside the curve $r = 3$.



$2 - 2\sin\theta = 3$

$-2\sin\theta = 1$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$ 1

1

$\text{Area} = \int_{7\pi/6}^{11\pi/6} \frac{1}{2}(2 - 2\sin\theta)^2\, d\theta - \int_{7\pi/6}^{11\pi/6} \frac{1}{2}(3)^2\, d\theta$ 1 1 1

5

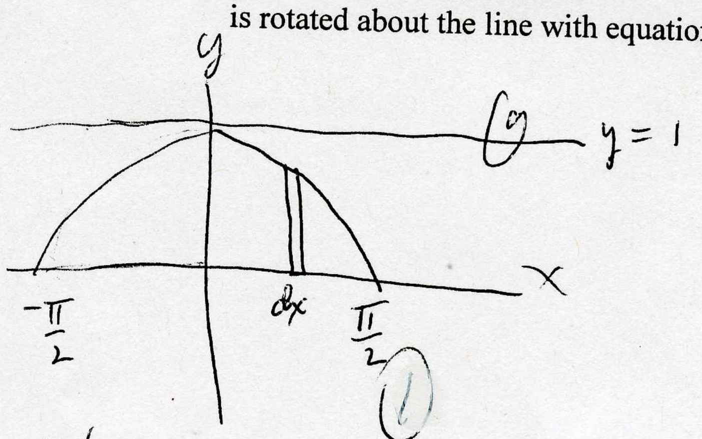
or $2\int_{7\pi/6}^{3\pi/2} \sim\sim\sim$ or $2\int_{3\pi/2}^{11\pi/6} \sim\sim\sim$

(c) Set up, but <u>DO NOT EVALUATE</u>, the integral representing the volume of the solid formed when the region bounded by

$y = \cos x$, $y = 0$, and $x = 0$

is rotated about the line with equation $y = 1$.

πr^2



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$V = \int_{-\pi/2}^{\pi/2} \pi(1)^2\, dx - \int_{-\pi/2}^{\pi/2} \pi(1 - \cos x)^2\, dx$ 1 1

1

4