MATH 1700: Test #5 (Fall 2008) Solutions

[7] 1. Write the general form (in terms of unknown coefficients) of the partial fraction decomposition (expansion) of the following expression. DO NOT SOLVE FOR THE COEFFICIENTS.

$$\frac{1}{(x^2 - 2x)(x^2 + 4)^2} =$$

Solution:
$$\frac{1}{(x^2 - 2x)(x^2 + 4)^2} = \frac{1}{x(x - 2)(x^2 + 4)^2} = \frac{A}{x} + \frac{B}{x - 2} + \frac{C_1 x + D_1}{x^2 + 4} + \frac{C_2 x + D_2}{(x^2 + 4)^2}$$

[9] **2.** Evaluate
$$\int_{2}^{\infty} \frac{1}{x^{1.001}} dx$$
.

Solution.

B1.

$$\int_{2}^{\infty} \frac{1}{x^{1.001}} dx = \lim_{t \to \infty} \int_{2}^{t} \frac{1}{x^{1.001}} dx = \lim_{t \to \infty} \left(\frac{x^{-0.001}}{-0.001} \right) \left| \begin{array}{c} t \\ 2 \end{array} \right| = -\frac{1}{0.001} \lim_{t \to \infty} \left(\frac{1}{t^{0.001}} - \frac{1}{2^{0.001}} \right) = \frac{1}{0.001} \frac{1}{2^{0.001}}$$

[8] 3. Set up but DO NOT EVALUATE the integral for the length of the part of the curve $y = -x^2 + 4$ not below the x-axis.

Solution. The graph of $y = -x^2 + 4$ is not below the x-axis for x in the interval [-2.2]. So the arc length we want is

$$\int_{-2}^{2} \sqrt{1 + (y')^2} \, dx = \int_{-2}^{2} \sqrt{1 + (-2x)^2} \, dx \, .$$