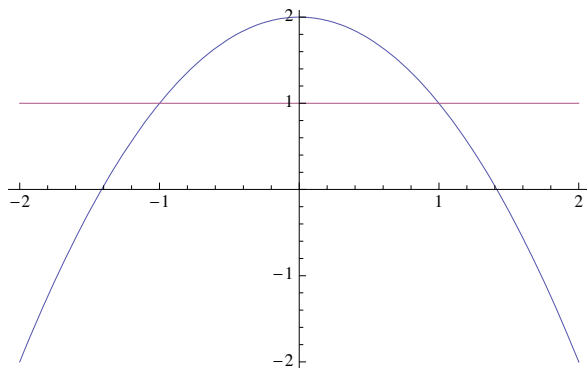


B3.

**MATH 1700: Test #3 (Fall 2008)**  
**Solutions**

- [9] 1. Find the area of the region bounded by the curves  $y = -x^2 + 2$  and  $y = 1$ . Sketch  $R$ .



*Solution:*

Solving  $y = -x^2 + 2$  and  $y = 1$  gives  $x = -1$  and  $x = 1$ . So, the area is  $\int_{-1}^1 [(-x^2 + 2) - 1] dx = 2 \int_0^1 (-x^2 + 1) dx = 2 \left( -\frac{x^3}{3} + x \right) \Big|_0^1 = 2 \left( 1 - \frac{1}{3} \right)$ .

- [8] 2. Find the area of bounded by the spiral  $r = \theta$ , the  $x$ -axis and the  $y$ -axis, for  $0 \leq \theta \leq \frac{\pi}{2}$ .

*Solution:* Area =  $\int_0^{\pi/2} \frac{1}{2} r(\theta)^2 d\theta = \int_0^{\pi/2} \frac{1}{2} \theta^2 d\theta = \frac{1}{2} \frac{\theta^3}{3} \Big|_0^{\pi/2} = \frac{1}{6} \left( \frac{\pi}{2} \right)^3$ .

- [9] 3. Set up, but **do not** evaluate, the integral for the volume of the solid we get by rotating the region bounded by the curves  $y = -x^2 + 2$  and  $y = 1$  around the  $x$ -axis.

*Solution. Volume* =  $\int_{-1}^1 [(-x^2 + 2)^2 \pi - 1^2 \pi] dx$ .