B2.

MATH 1700: Test #2 (Fall 2008)
Solutions

[8] **1.** Use the definition only to express the definite integral $\int_0^2 (x^3+1)dx$ as a limit of a specific sum. Do not compute that limit (or the integral). [For example, we figured out in class that $\int_0^1 x^2 dx = \lim_{n\to+\infty}\sum_{i=1}^{n}\left(i\frac{1}{n}\right)^2\frac{1}{n}$; you should go that far for Question 1.]

Solution: Subdividing the interval [0,2] into *n* many parts gives intervals of size $\frac{2}{n}$ each. The right hand edges of these intervals are $\frac{2}{n},\ 2\frac{2}{n},\ 3\frac{2}{n},\ \ldots,\ i\frac{2}{n},\ \ldots,\ n\frac{2}{n}$. (Note: choosing left-hand side edges, or any other points in the small intervals is also correct) So,

$$\int_0^2 (x^3+1)dx = \lim_{n\to+\infty}\sum_{i=1}^{n}\left[\left(i\frac{2}{n}\right)^3+1\right]\frac{2}{n}.$$

[7] **2.** Evaluate:

(a) $\frac{d}{dx}\int_0^1 \ln(1+e^t)\,dt$

(b) $\frac{d}{dx}\int_0^{\sin x} \ln(1+e^t)\,dt$

Solution. **(a)** $\frac{d}{dx}\int_0^1 \ln(1+e^t)\,dt = 0$, since $\int_0^1 \ln(1+e^t)\,dt$ is a constant.

(b) By the Fundamental Theorem of Calculus, and by the chain rule, we have:

$$\frac{d}{dx}\int_0^{\sin x} \ln(1+e^t)\,dt = [\ln(1+e^{\sin x})]\cos x$$

[9] **3.** Evaluate the following integrals:

(a) $\int_{-1}^{1}\frac{x^3}{x^{100}+1}\,dx$

(b) $\int_0^1 e^x\sqrt{1+e^x}\,dx$

Solution. (a) The function $f(x)=\frac{x^3}{x^{100}+1}$ is odd, since $f(-x)=\frac{(-x)^3}{(-x)^{100}+1}=\frac{-x^3}{x^{100}+1}=-f(x)$.

Because of that, and since the limits of integration are mutual negatives, $\int_{-1}^{1}\frac{x^3}{x^{100}+1}\,dx=0$.

(b) Use the substitution $u=1+e^x$, so that $du=e^x dx$. With that we have

$$\int_0^1 e^x\sqrt{1+e^x}\,dx=\int_{x=0}^{x=1}\sqrt{u}\,du=\frac{2}{3}u^{\frac{3}{2}}\Big|_{x=0}^{x=1} = \frac{2}{3}(1+e^x)^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3}(1+e)^{\frac{3}{2}}-\frac{2}{3}(1+1)^{\frac{3}{2}}.$$