B1.

MATH 1700: Test #1 (Fall 2008)
Solutions

1. Evaluate the following limits:

(a) $\lim_{x\to 0}\frac{x^2+x}{e^x}$

(b) $\lim_{x\to\infty}\frac{x^2+x}{e^x}$

Solution: **(a)** $\lim_{x\to 0}\frac{x^2+x}{e^x}=0$.

(b) $\lim_{x\to\infty}\frac{x^2+x}{e^x}\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}}\lim_{x\to\infty}\frac{2x+1}{e^x}\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}}\lim_{x\to\infty}\frac{2}{e^x}=0$.

2. Find $\frac{d^2y}{dx^2}$ at the point when $t=1$ if $x=3+t$ and $y=t^2-t^4$.

Solution: $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2t-4t^3}{1}$; $\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}=\frac{2-12t^2}{1}=2-12t^2$. So, at $t=1$ we get $\frac{d^2y}{dx^2}=2-12=-10$.

3. Find the equation of the tangent line to the curve $r=1+2\cos\theta$ (in polar coordinates) at the point when $\theta=\frac{\pi}{2}$.

Solution: $x=r\cos\theta=(1+2\cos\theta)\cos\theta$, $y=r\sin\theta=(1+2\cos\theta)\sin\theta$. At $\theta=\frac{\pi}{2}$ we compute $x=0$ and $y=1$.

Further: $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\cos\theta-2\sin^2\theta+2\cos^2\theta}{-\sin\theta-4\cos\theta\sin\theta}$. When $\theta=\frac{\pi}{2}$ we find $\frac{dy}{dx}=\frac{-2}{-1}=2$. So, the tangent we want is $y-1=2x$.