B1.   **MATH 1700: Test #5**
                            Solutions

  1. Write the general form (in terms of unknown coefficients) of the partial fractions expansion of each of the following expressions. DO NOT SOLVE FOR THE COEFFICIENTS.

(a) $\dfrac{1}{x^2 - x - 2}$

(b) $\dfrac{x}{(x-2)(x^2+2)^2}$

Solution. (a) $\dfrac{1}{x^2-x-2} = \dfrac{1}{(x+1)(x-2)} = \dfrac{A}{x+1} + \dfrac{B}{x-2}$

(b) $\dfrac{x}{(x-2)(x^2+2)^2} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+2} + \dfrac{Dx+E}{\left(x^2+2\right)^2}$

2.   Evaluate $\displaystyle\int_1^\infty \dfrac{2}{1+x^2}\,dx$ or show it diverges.

Solution.

$$\int_1^\infty \frac{2}{1+x^2}\,dx = \lim_{t\to\infty}\int_1^t \frac{2}{1+x^2}\,dx = \lim_{t\to\infty}\left(2\tan^{-1}x\Big|_1^t\right) = \lim_{t\to\infty}\left(2\tan^{-1}t - 2\tan^{-1}1\right) = 2\frac{\pi}{2} - 2\frac{\pi}{4}.$$

3. Set up the integral for the arc length of the curve $y = 3\ln x$ from $x = 2$ to $x = 5$. DO NOT EVALUATE the integral.

Solution. $\displaystyle\int_2^5 \sqrt{1 + \frac{9}{x^2}}\,dx$.