B4.
MATH 1700: Test #4
Solutions

[8]   **1.** Evaluate $\int x^2 \ln x \ dx$

Solution. Use integration by parts, with $u = \ln x$, $dv = x^2 dx$. We compute: $du = \dfrac{dx}{x}$ and

$v = \dfrac{x^3}{3}$. So, $\int x^2 \ln x \ dx = \dfrac{x^3}{3} \ln x - \int \dfrac{x^3}{3} \dfrac{1}{x} \ dx = \dfrac{x^3}{3} \ln x - \dfrac{x^3}{9} + c$.

[8]   **2.** Evaluate $\int 2\sin x \cos^2 x \ dx$

Solution.
Use $u = \cos x$, so that $du = -\sin x \ dx$:

$$\int 2\sin x \cos^2 x \ dx = 2\int -u^2 du = -2\dfrac{u^3}{3} + c = -2\dfrac{\cos^3 x}{3} + c.$$

[9]   **3.** Evaluate $\int x^3 \sqrt{x^2 - 1} \ dx$

Solution.
Use $u = x^2 - 1$, so that $du = 2x \ dx$, and $x^2 = u + 1$:

$$\int x^3 \sqrt{x^2 - 1} \ dx = \int (u+1)\sqrt{u} \ \dfrac{du}{2} = \dfrac{1}{2}\int (u^{3/2} + u^{1/2})du =$$

$$= \dfrac{1}{2}(\dfrac{u^{5/2}}{5/2} + \dfrac{u^{3/2}}{3/2}) + c = \dfrac{(x^2 - 1)^{5/2}}{5} + \dfrac{(x^2 - 1)^{3/2}}{3} + c \qquad .$$