

B3.

MATH 1700: Test #3
Solutions

[12] 1. A solid stretches from $x = 0$ to $x = 3$. The cross-sections of that solid with planes perpendicular to the x -axis and x many units away from the origin are squares of edges $2x$ units long. Find the volume of that solid.

Solution. Since each cross-section is a square of side $2x$ its area is $A(x) = 4x^2$. So, the volume of the solid is $\int_0^3 A(x)dx = \int_0^3 4x^2 dx = 4 \frac{x^3}{3} \Big|_0^3 = 36$ cubic units.

[13] 2.

[6] (a) Differentiate $x \sin^{-1}(2x)$.[7] (b) Evaluate $\int \frac{e^x}{1 + e^{2x}} dx$.**Solution.**

$$(a) \left(x \sin^{-1}(2x) \right)' = \sin^{-1}(2x) + x \frac{1}{\sqrt{1-x^2}} 2$$

(b) Use the substitution $u = e^x$, so that $du = e^x dx$:

$$\int \frac{e^x}{1 + e^{2x}} dx = \int \frac{1}{1 + u^2} du = \tan^{-1} u + c = \tan^{-1} e^x + c.$$