B2.

MATH 1700: Test #2
Solutions

1. (a) Use the definition of definite integral to write $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\sin(\frac{i}{n}+1)$ as a definite integral. (Do not justify anything here; just write the answer!)

(b) Use part (a) and the Fundamental Theorem of Calculus to evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\sin(\frac{i}{n}+1)$.

Solution. (a) $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\sin(\frac{i}{n}+1)=\int_0^1 \sin(x+1)dx$

(b) $\int_0^1 \sin(x+1)dx = -\cos(x+1)\Big|_0^1 = -\cos 2+\cos 1$.

2. Evaluate the following integrals

(a) $\int \frac{(\ln x)^3}{x}dx$

(b) $\int_0^1 xe^{-x^2}dx$

Solution.

(a) Use the substitution $u=\ln x$ and compute $du=\frac{1}{x}dx$. So

$$\int \frac{(\ln x)^3}{x}dx=\int u^3 du=\frac{u^4}{4}+c=\frac{(\ln x)^4}{4}+c$$

(b) Use the substitution $u=-x^2$ and compute $du=-2x\ dx$.

$$\int_0^1 xe^{-x^2}dx=\int_{x=0}^{x=1}(-\frac{1}{2})e^u du=(-\frac{1}{2})e^u\Big|_{x=0}^{x=1}=-\frac{1}{2}e^{-x^2}\Big|_0^1=-\frac{1}{2}e+\frac{1}{2}.$$