B1.

MATH 1700: Test #1
Solutions

1. We are given the curve $x = t^3 - 3t$, $y = \frac{1}{2}t^2 - 2t$.

(a) Find $\frac{dy}{dx}$ at the point when $t = 0$.

Solution. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 3}{t - 2}$, and when $t = -1$ we get $\frac{dy}{dx} \overset{at\ t=0}{=} \frac{3}{2}$.

(b) Find all of the points where the tangent lines are vertical or horizontal. (Note: finding a point means finding the coordinates of that point).

Solution. For the tangents to be horizontal it is necessary that $\frac{dy}{dt} = 0$. This happens when $3t^2 - 3 = 0$ which gives $t = 1$ or $t = -1$. The moment $t = 1$ yields the point $(-2, -\frac{3}{2})$, while $t = -1$ yields $(2, \frac{5}{2})$.

For the tangents to be vertical it is necessary that $\frac{dx}{dt} = 0$. This happens when $t - 2 = 0$ which gives $t = 2$. This gives the point $(2, -2)$

[Note that there is no point where both $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} = 0$, and so the above points are indeed points where the tangents are horizontal or vertical respectively.]

2. (a) Find some polar coordinates of the point $(2, 2)$.

Solution. $r = \sqrt{2^2 + 2^2} = \sqrt{8}$. $\tan\theta = \frac{2}{2}$, and one solution is $\theta = \frac{\pi}{4}$. So, one pair of polar coordinates for the given point is $(\frac{\pi}{4}, \sqrt{8})$.

(b) Find the Cartesian coordinates of the point $(\frac{\pi}{3}, 2)$ given in polar coordinates. (I write the polar angle as the first coordinate, the polar distance as the second coordinate.)

Solution. $x = r\cos\theta = 2\cos\frac{\pi}{3} = 1$, $y = r\sin\theta = 2\sin\frac{\pi}{3} = \sqrt{3}$, and so the Cartesian coordinates of the given point are $(1, \sqrt{3})$.

(c) Sketch the region defined by the inequalities $2 < r \le 3$ (where r is polar distance).

Solution. Figure to the right: the ring between two circles centered at the origin, the smaller of radius 2, the larger of radius 3. The points on the smaller circle are not included, while the points on the larger circle are included.