1 **136.151: Test #4**
**20 minutes**

**Name:____________________** **Student Number: _______**

**1.** Find the absolute maximum and the absolute minimum of the function $f(x) = x^3 - 3x^2 + 1$ over the interval $[1,4]$. Show your work (that is, justify your answer).

**Solution**. $f'(x) = 3x^2 - 6x$, and so the critical points come from solving $3x^2 - 6x = 0$, that is, $3x(x-2) = 0$. The solutions are obviously $x = 0$ and $x = 2$. The former is out of the interval we consider, so we ignore it.

At the critical points: $f(2) = -3$

At the edges: $f(1) = -1$, $f(4) = 17$

So, we have the absolute maximum of 17 when $x = 4$ and the absolute minimum of $-3$ when $x = 2$.

**2.** The perimeter if a rectangle is 40. Find the dimensions of the rectangle such that its area is as large as possible. Justify your answer.

**Solution.** If the dimensions are denoted by *x* and *y*, then the perimeter is $P = 2x + 2y$ and the area is $A = xy$. Since $P = 40$ we have that $40 = 2x + 2y$, from where we find that $y = 20 - x$. So, the area is $A = x(20 - x) = 20x - x^2$. We compute $A' = 20 - 2x$, and so the only critical point is $x = 10$. Since $A'' = -2 < 0$ the critical point yields a local maximum. Since it is the only critical point, this local maximum must be the absolute maximum. So, the desired dimensions are $x = 10$ and $y = 20 - 10 = 10$.

**3.** Suppose $y = -3x + 4$ and suppose $y$ is increasing at the rate of $3\ m/\sec$. Find the rate of change of *x*. Show your work (that is, justify your answer).

**Solution.** Differentiate with respect to time to get $\frac{dy}{dt} = -3\frac{dx}{dt}$. We are given that $\frac{dy}{dt} = 3\ m/\sec$ and so $3 = -3\frac{dx}{dt}$, from where we find that $\frac{dx}{dt} = -1\ m/\sec$.