2,3,4

136.151: Test #3
Solutions

Name:____________________ **Student Number:** _______

1. Find $\frac{dy}{dx}$ at (1,1) if $xy^3 + x^2 y = 2$ defines *y* as a function on *x*.

Solution. Differentiate the equation to get $y^3 + x(3y^2)\frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} = 0$. Substitute $x = 1$ and $y = 1$ to get $1 + 3\frac{dy}{dx} + 2 + \frac{dy}{dx} = 0$, from where we find that $\frac{dy}{dx} = -\frac{3}{4}$.

2. Find all critical points of the function $f(x) = x^3 + 3x^2 + 3$, then classify them using the second derivative test.

Solution. $f'(x) = 3x^2 + 6x$. Solving $3x^2 + 6x = 0$ gives $x = 0$ or $x = -2$. These are the critical points. Compute $f''(x) = 6x + 6$, and so $f''(0) = 6 > 0$ (local minimum), $f''(-2) = -6 < 0$ (local maximum).

3. Find all inflection points of the function $f(x) = x^4 + x^3$. Justify your answer.

Solution. Compute $f'(x) = 4x^3 + 3x^2$, and $f''(x) = 12x^2 + 12x = 12x(x+1)$. Solve to get $x = 0$ or $x = -1$. These are the only potential inflection points.
Taka a point to the left of –1; say at $x = -2$, $f''(-2) > 0$; so there, $f(x)$ is concave up ($\bigcup$).
Take a point between 0 and 1, say $\frac{1}{2}$ and notice that $f''(-1/2) < 0$; so $f(x)$ is concave down ($\bigcap$) there.
Take a point to the right of 0, say 100, and notice that $f''(100) > 0$; so $f(x)$ is concave up ($\bigcup$) there.
Since the function does change concavity at $x = 0$ and $x = -1$, these are indeed inflection points.