2,3,4

136.151: Test #1 Solutions
20 minutes

Name:_____________________ **Student Number:** _______

1. Consider the following three lines.

$l_1: -2y = -2x + 1$
$l_2: -y + x = 10$
$l_3: y = -2 - x$

Find the slopes of each of the lines. Which of these lines are mutually parallel, which are mutually perpendicular? Why?

Solution.

For l_1 we find that $y = x - \frac{1}{2}$ so that the slope of that line is $m_1 = 1$.

For l_2 we find that $y = x - 10$ so that the slope of that line is $m_2 = 1$.

For l_3 we are given that $y = -x - 2$ so that the slope of that line is $m_3 = -1$.

Since $m_1 = m_2$, the first two lines are parallel. Since $m_1 = -\frac{1}{m_3}$, the first and the third line are perpendicular. Consequently, so are the second and the third.

2. Use interval notation to describe the domain of the function $\frac{\sqrt{x^2+1}}{\sqrt{1-x}}$

Solution.

The denominator should not be 0, so that $1 - x \neq 0$, which tell us that $x \neq 1$. The expression inside the root in the denominator must be ≥ 0, i.e., $1 - x \geq 0$, i.e., $x \leq 1$. Together with the first sentence we have that $x < 1$. The expression inside the root in the numerator must also be at ≥ 0; but, because $x^2 \geq 0$, that is true always for all numbers x. So the root in the numerator places no restrictions on x. Summarizing, the domain of the function is $(-\infty, 1)$.

3. (a) Is the function $f(x) = \frac{x|x|}{1+2x^2}$ odd, even or neither? Why?

(b) Give an example of a function that is both even and odd.

(c) Show that for every two odd functions $f(x)$ and $g(x)$ their product $h(x) = f(x)g(x)$ is an even function.

Solution.

(a) $f(-x) = \frac{(-x)|-x|}{1+2(-x)^2} = \frac{-x|x|}{1+2x^2} = -f(x)$, and so the function is odd.

(b) One such example is the function $f(x) = 0$.

(c) $h(-x) = f(-x)(g(-x) = (-f(x))(-g(x)) = f(x)g(x) = h(x)$.