B04.

136.150: Test #5
20 minutes
No Calculators

Name:_____ **Student Number:** _____

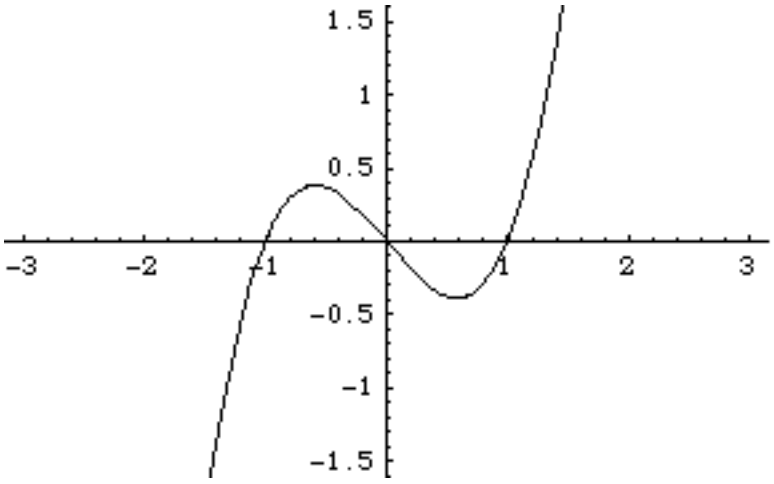
1. Consider $f(x) = x^3 - x$.

[9] **(a)** Find and classify the local extrema .

[7] **(b)** Sketch the graph of the function.

Solution. (a) Since $f'(x) = 3x^2 - 1$ the critical points are $x = \frac{-1}{\sqrt{3}}$ and $x = \frac{1}{\sqrt{3}}$. Since $f''(x) = 6x$ the second derivative test gives the first point as a local maximum, the second as a local minimum.

(b)



[9] **2.** Suppose $f''(x) = x$, $f'(1) = 1$ and $f(1) = 1$. Find $f(x)$.

Solution. $f'(x) = \frac{x^2}{2} + c_1$, and $f(x) = \frac{x^3}{6} + c_1 x + c_2$. From the initial conditions we find $1 = \frac{1}{2} + c_1$ and $1 = \frac{1}{6} + c_1 + c_2$. Solving this gives $c_1 = \frac{1}{2}$ and $c_2 = \frac{1}{3}$. So, $f(x) = \frac{x^3}{6} + \frac{1}{2}x + \frac{1}{3}$.