B07.

136.150: Test #4
20 minutes
Solutions

1. Find y' if $y = x^{2x}$.

Solution. If $y = x^{2x}$ then $\ln y = 2x \ln x$. Differentiate this to get $\frac{y'}{y} = 2\ln x + 2$, from where we find that $y' = y(2\ln x + 2) = x^{2x}(2\ln x + 2)$.

2. Find all local extrema of the function $f(x) = x - x^2 + 3$. Use the second derivative test to classify these local extrema (as maxima or minima). (No points will be awarded if the second derivative test is not used).

Solution. $f'(x) = 2 - 2x$ and $f'(x) = 0$ gives $x = 1$. This is the only critical point. Further, $f''(x) = -2$ and since this is always less than 0 it follows that our critical point yields a local maximum.

3. Find the interval(s) where the graph of the function $f(x) = x^2 - x^3$ is concave up.

Solution. Compute $f'(x) = 2x - 3x^2$ and $f''(x) = 2 - 6x$. The function $f(x) = x^2 - x^3$ is concave up where $f''(x) > 0$. Solving $f''(x) = 2 - 6x > 0$ yields $x < \frac{1}{3}$. So, the function is concave up over the interval $(-\infty, \frac{1}{3})$.