B05.

136.150: Test #2
Solutions

1. Find the limit or, if it does not exist, check if it is ∞, $-\infty$ or neither. Show your work.

(a) $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

(b) $\lim_{x\to\infty}\frac{\sqrt{x}+1}{x-1}$

Solution. **(a)** $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}=\lim_{x\to 1}\frac{1}{\left(\sqrt{x}+1\right)}=\frac{1}{2}$

(b)

$$\lim_{x\to\infty}\frac{\sqrt{x}+1}{x-1}=\lim_{x\to\infty}\frac{\sqrt{x}\left(1+\frac{1}{\sqrt{x}}\right)}{\sqrt{x}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)}=\lim_{x\to\infty}\frac{\left(1+\frac{1}{\sqrt{x}}\right)}{\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)}=0$$

since the denominator in the last fraction tens to infinity while the numerator tends to 1.

2. Indicate which of the following functions are **NOT** continuous at the point when x=2. You do **NOT** need to justify your answers.

$$f(x)=\frac{x-2}{x-2},\quad g(x)=\frac{x-2}{x+2},\quad h(x)=\begin{cases}x-2 & \text{if } x<2\\ 0 & \text{if } x\geq 2\end{cases},\quad k(x)=\begin{cases}x+2 & \text{if } x<2\\ 0 & \text{if } x\geq 2\end{cases}$$

Solution. $f(x)$ and $k(x)$ are not continuous.
($f(x)$ since $f(2)$ does not exist, and $k(x)$ since the one-sided limits at x =2 are distinct.)

3. Find the slope of the tangent line to the curve defined by $f(x)=1-x^2$ at the point when x = 1. Show your work. (You may use only the theory from the material covered by this and the first quiz. Only the slope is needed, **not** the full equation of the tangent line).

Solution. The required slope is the limit $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$. We compute:

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{1-(1+h^2)-(1-1^2)}{h}=\lim_{h\to 0}\frac{h^2}{h}=\lim_{h\to 0}h=0\,.$$