B04.

136.150: Test #1
20 minutes

Name:_____________________ **Student Number:** _______

1. Find the domain of the following function. State your answer in terms of intervals.

$f(x) = \sqrt{1-2x}$

Solution. Because of the root we must have $1-2x \geq 0$. We solve it:
$1-2x \geq 0 \Leftrightarrow 1 \geq 2x \Leftrightarrow \frac{1}{2} \geq x$ where the symbol $\Leftrightarrow$ means "equivalent to", or "means the same as". The last inequality tells us that the domain of this function is the semi-open interval $[\frac{1}{2}, \infty)$.

2. Which of the following functions is even, which is odd, which is neither. Justify your answers using the definitions of odd/even functions.

$$f(x) = x^4 - x,\ g(x) = \frac{1}{x^2-1},\ h(x) = x - 2x^3$$

Solution. $f(-x) = (-x)^4 - (-x) = x^4 + x$, and this is neither $f(x)$ nor $-f(x)$. So, this function is neither even nor odd.

$g(-x) = \frac{1}{(-x)^2 - 1} = \frac{1}{x^2-1}$, so this is an even function.

$h(-x) = (-x) - 2(-x)^3 = x + 2x^3$ which is (easy to see) equal to $-h(x)$. So, this is an odd function.

3. Find $\lim_{x\to 3^+} f(x)$ and $\lim_{x\to 3^-} f(x)$ for $f(x)$ given below. Does $\lim_{x\to 3} f(x)$ exist? (Justify your answers).

$$f(x) = \begin{cases} 2x-5 & \text{if } x > 3 \\ 2-x & \text{if } x < 3 \end{cases}$$

Solution. $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x - 5 = 1$, where we have used the fact that $f(x)$ is $2x-5$ when $x > 3$ (which is what happens in this limit).

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 2 - x = -1$, where we have used the fact that $f(x)$ is $2-x$ when $x < 3$ (which is what happens in this limit).

Since the two one sided limits (that we have computed above) are not equal, it follows (by a property we have covered) that $\lim_{x\to 3} f(x)$ does not exist.