B11.

136.150: Test #3 Solutions

Name:____________________ **Student Number:** _______

[6] **1.** Find $f'(x)$ if $f(x)=(\sin x)^{\cos x}$

Solution. Set $y=\sin x^{\cos x}$, so that $\ln y=\ln\left(\sin x^{\cos x}\right)$, i.e., $\ln y=(\cos x)\ln(\sin x)$. Differentiate with respect to x to get $\frac{1}{y}y'=(-\sin x)\ln(\sin x)+(\cos x)\frac{1}{\sin x}(\cos x)$, and thus

$$y'=(\sin x)^{\cos x}[(-\sin x)\ln(\sin x)+(\cos x)\frac{1}{\sin x}(\cos x)]$$

[7] **2.** Find all critical points of the function $f(x)=x^3-3x^2$ and then classify them using the second derivative test.

Solution. $f'(x)=3x^2-6x$; $f'(x)=0$ gives $3x^2-6x=0$ or $3x(x-2)=0$ which yields $x=0$ or $x=2$. These are the only two critical points (values). Computing further gives $f''(x)=6x-6$ and so $f''(0)=-6<0$ and $f''(2)=6>0$. Hence we have a local maximum at $x=-6$ and a local minimum at $x=6$.

[7] **3.** Find the intervals where the function $f(x)=-xe^x$ is concave up and where it is concave down.

Solution. We compute $f'(x)=-e^x-xe^x$ and $f''(x)=-e^x-e^x-xe^x=e^x(-2-x)$. The only potential inflection point is $x=-2$. Choose, say, $x=-3$ in $(-\infty,-2)$ and notice that $f''(-3)=e^x(-2+3)>0$. So the function is concave up over $(-\infty,-2)$. Choose, say, $x=0$ in $(-2,\infty)$ and notice that $f''(0)=e^x(-2)<0$. So the function is concave down over $(-2,\infty)$.