B10.

136.150: Test #3 Solutions

Name:______________________ **Student Number:** _______

[8] **1.** Find $f'(x)$. Do not simplify your answer after differentiating.

(a) $f(x) = \dfrac{\sqrt{x}}{\sin x + \cos x}$

(b) $f(x) = (1 + \tan(1-x))^{-2}$

Solution. (a)

$$f'(x) = \frac{\left(\sqrt{x}\right)'(\sin x + \cos x) - \sqrt{x}(\sin x + \cos x)'}{(\sin x + \cos x)^2} = \frac{\frac{1}{2\sqrt{x}}(\sin x + \cos x) - \sqrt{x}(\cos x - \sin x)}{(\sin x + \cos x)^2}.$$

(b) $f'(x) = (-2)(1 + \tan(1-x))^{-1}\left(\dfrac{1}{\cos^2(1-x)}\right)(-1)$

[6] **2.** Find y' at the point (1,1) if $y^2 + xy = 2$.

Solution. $\dfrac{d}{dx}(y^2 + xy) = \dfrac{d}{dx}(2)$; so $2y\dfrac{dy}{dx} + y + x\dfrac{dy}{dx} = 0$. At the given point we have $x = 1$ and $y = 1$, so that $2\dfrac{dy}{dx} + 1 + \dfrac{dy}{dx} = 0$, from where we find that $\dfrac{dy}{dx} = -\dfrac{1}{3}$.

[6] **3.** Suppose the sides of a square are increasing in such a way that the area of the square is increasing at the rate of $4m/\sec$. How fast is the side of the square increasing at the moment when the area is $25m^2$? Justify your answer.

Solution. A = area of the square; x=the length of a side of the square. Then $A = x^2$. Differntiate with respect to time t to get $\dfrac{dA}{dt} = 2x\dfrac{dx}{dt}$. At the given moment the area is $25m^2$ and so the side is $5m$. Since $\dfrac{dA}{dt}$ is fixed to $4m/\sec$, we have $4 = 2(5)\dfrac{dx}{dt}$ from where we find that $\dfrac{dx}{dt} = \dfrac{2}{5}m/\sec$, i.e., x is increasing at the rate of $\dfrac{2}{5}m/\sec$.