B09.

136.150: Test #2 Solutions

Name:____________________ **Student Number:** _______

[6] **1.** Find all horizontal asymptotes of the function $f(x)=\dfrac{\sqrt{x^2+1}}{x-1}$.

Solution. $\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+1}}{x-1}=\lim\limits_{x\to\infty}\dfrac{x+1}{x-1}=\lim\limits_{x\to\infty}\dfrac{x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{1}{x}\right)}=1$. So $y=1$ is a horizontal asymptote as $x\to\infty$.

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{x-1}=\lim\limits_{x\to-\infty}\dfrac{-x+1}{x-1}=\lim\limits_{x\to-\infty}-1=-1$. So $y=-1$ is a horizontal asymptote as $x\to-\infty$.

[7] **2.** **(a)** What does it mean to say that a function $f(x)$ is continuous at $x=2$? (Write down the definition.)

(b) Find the value of the constant *c* so that the function $f(x)=\begin{cases} cx-2 & \text{if } x>2 \\ x^2-2cx & \text{if } x\le 2\end{cases}$ is continuous at $x=2$. (Justify your answer using one-sided limits.)

Solution. **(a)** $f(2)=\lim\limits_{x\to 2} f(x)$.

(b) $f(2)=2^2-(2c)2=4-4c$

$\lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+} cx-2=2c-2$

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-} x^2-2cx=4-4c$. For the function to be continuous all of these 3 numbers should be equal, which gives $2c-2=4-4c$. Solve this to get $c=1$.

[7] **3.** **(a)** Compute $f'(x)$ if $f(x)=5\sqrt[5]{x}+\dfrac{4}{x^2}$

(b) Compute $g'(x)$ if $g(x)=\dfrac{\sqrt{x}-x}{x^2}$ (Note: all computations should be based on the material covered for this test; in particular, the "quotient rule" may not be used).

Solution. **(a)** $f'(x)=\left(5\sqrt[5]{x}+\dfrac{4}{x^2}\right)'=\left(5x^{\frac{1}{5}}\right)'+\left(4x^{-2}\right)'=5\dfrac{1}{5}x^{-\left(\frac{4}{5}\right)}+4(-2)x^{-3}=x^{-\left(\frac{4}{5}\right)}-8x^{-3}$.

(b) $g(x)=\dfrac{\sqrt{x}-x}{x^2}=\dfrac{x^{\frac{1}{2}}-x}{x^2}=\dfrac{x^{\frac{1}{2}}}{x^2}-\dfrac{x}{x^2}=x^{-\left(\frac{3}{2}\right)}-x^{-1}$. So,

$g'(x)=\left(x^{-\left(\frac{3}{2}\right)}-x^{-1}\right)'=-\dfrac{3}{2}x^{-\left(\frac{5}{2}\right)}+x^{-2}$