B08.

136.150: Test #1
Solutions

Name:_____________________ **Student Number:** _______

1. Let $f(x)=\sqrt{x-1}$ and let $g(x)=2x$
(a) Find the composition $f\circ g(x)$
(b) Find the domain of the function $f\circ g(x)$.

Solution. (a) $f\circ g(x)=f(g(x))=f(2x)=\sqrt{2x-1}$.
(b) Because of the root we must have $2x-1\geq 0$. We solve it:
$2x-1\geq 0 \Leftrightarrow 2x\geq 1 \Leftrightarrow x\geq \frac{1}{2}$ where the symbol $\Leftrightarrow$ means "equivalent to", or "means the same as". The last inequality tells us that the domain of this function is the semi-open interval $[{}^1\!/_2,\infty)$.

2. Find all vertical asymptotes of the function $f(x)=\frac{1}{x-1}$. (Do not forget to justify your answer by means of certain limits.)

Solution. The domain of the function is obviously $(-\infty,1)$ together with $(1,\infty)$. So, the only potential vertical asymptote is $x=1$. We compute
$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\frac{1}{x-1}=+\infty$. This tells us that $x=1$ is indeed a vertical asymptote. (So does the following: $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\frac{1}{x-1}=-\infty$.)

3. Compute each of the following limits, or show it does not exist.
(a) $\lim_{x\to -2}\frac{2-x}{8-x^3}$
(b) $\lim_{x\to 2}\frac{2-x}{8-x^3}$

Solution. (a) $\lim_{x\to -2}\frac{2-x}{8-x^3}=\frac{4}{16}=\frac{1}{4}$
(b) $\lim_{x\to 2}\frac{2-x}{8-x^3}=\lim_{x\to 2}\frac{2-x}{(2-x)(4+2x+x^2)}=\lim_{x\to 2}\frac{1}{(4+2x+x^2)}=\frac{1}{12}$.