# Midterm Exam 136.150
# BRIEF Solutions

**Values**

[11] 1.

(a)

$$\lim_{x\to 3}\frac{1}{x-3}-\frac{6}{x^2-9}=\lim_{x\to 3}\frac{x+3-6}{(x-3)(x+3)}=\lim_{x\to 3}\frac{x-3}{(x-3)(x+3)}=\lim_{x\to 3}\frac{1}{x+3}=\frac{1}{6}$$

(b)

$$\lim_{x\to 1}\frac{x^2-4x+3}{x^2-1}=\lim_{x\to 1}\frac{(x-3)(x-1)}{(x-1)(x+1)}=\lim_{x\to 1}\frac{x-3}{x+1}=\frac{-2}{2}=-1$$

(c)

$$\lim_{x\to\infty}(\sqrt{x^2+x}-x)=\lim_{x\to\infty}(\sqrt{x^2+x}-x)\frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x^2+x-x^2}{\sqrt{x^2+x}+x}=$$

$$=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}=\lim_{x\to\infty}\frac{x}{x\sqrt{1+\frac{1}{x}}+x}=\lim_{x\to\infty}\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}=\lim_{x\to\infty}\frac{1}{\left(\sqrt{1+\frac{1}{x}}+1\right)}=\frac{1}{2},$$

[10] 2. Since the function $f(x)$ is a polynomial over the intervals $(-\infty,-1)$, $(-1,3)$ and $(3,+\infty)$, it is continuous at all points in these intervals.

In order $f(x)$ to be continuous at $x=-1$ it has to be that $\lim_{x\to -1}f(x)=f(-1)$. We compute:

$f(-1)=2$

$\lim_{x\to -1^-}f(x)=2$

$\lim_{x\to -1^+}f(x)=-a+b$

and so it must be that $-a+b=2$.

In order $f(x)$ to be continuous at $x=3$ it has to be that $\lim_{x\to 3}f(x)=f(3)$. We compute:

$f(3)=-2$

$\lim_{x\to 3^-}f(x)=3a+b$

$\lim_{x\to -1^+}f(x)=-2$

and so it must be that $3a+b=-2$.

Solving $-a+b=2$ and $3a+b=-2$ gives $a=-1$ and $b=1$.

So, the function $f(x)$ is continuous everywhere if $a=-1$ and $b=1$.

[14] 3.

(a) $(4\sqrt[3]{x}+\sec x+\frac{1}{x^2}+\sin 2)'=\frac{4}{3}x^{-\frac{2}{3}}+\frac{\sin x}{\cos^2 x}-2x^{-3}$

(b) $\left(\frac{x}{x^2+\cos x}\right)'=\frac{(x^2+\cos x)-x(2x-\sin x)}{(x^2+\cos x)^2}$

(c) $\left(e^{x\tan x}\right)'=e^{x\tan x}(\tan x+\frac{x}{\cos^2 x})$

[6] 4.

$$\left(\frac{1}{x+3}\right)' = \lim_{h\to 0}\frac{\frac{1}{x+h+3}-\frac{1}{x+3}}{h} = \lim_{h\to 0}\frac{x+3-(x+h+3)}{h(x+h+3)(x+3)} = \lim_{h\to 0}\frac{-h}{h(x+h+3)(x+3)} =$$

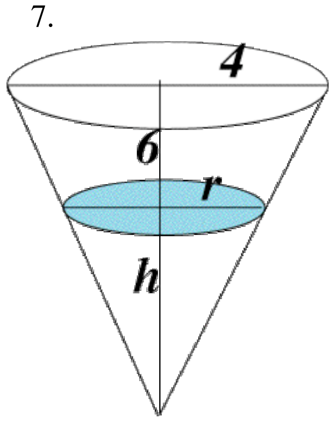
$$= \lim_{h\to 0}\frac{-1}{1(x+h+3)(x+3)} = \frac{-1}{(x+3)^2}$$

[7] 5. $\left(2e^{-x}+e^{y}\right)' = \left(3e^{x-y}\right)'$

$-2e^{-x}+e^{y}y' = 3e^{x-y}(1-y')$; now substitute $x=0$ and $y=0$ to get $-2+y' = 3(1-y')$ from where we find that $y' = \frac{5}{4}$. This is the slope of the tangent line. Since the *y*-intercept is given to be 0, the tangent line is $y = \frac{5}{4}x$.

[4] 6. Proven in class and in the textbook.

[8] 7.




V is the volume of the water in the cup; *h* is the height of the water in the cup; *r* is the radius of the surface of the water in the cup. We are given that $\frac{dV}{dt} = -2$, where $V = \frac{r^2 h\pi}{3}$. We see from the picture that $\frac{r}{h} = \frac{4}{6}$ from where we find that $r = \frac{2}{3}h$. Substitute this in the formula for the volume, get $V = \frac{4h^3\pi}{27}$. Differentiate this with respect to time (and recall that $\frac{dV}{dt} = -2$), get $-2 = \frac{4(3h^2)\pi}{27}\frac{dh}{dt}$. At the given moment $h=3$; substitute that in the last equation and solve for $\frac{dh}{dt}$ to get $\frac{dh}{dt} = -\frac{1}{2\pi}$. So, the level of the water is dropping at the rate $\frac{1}{2\pi}\ cm/\min$.