B04. **MATH 1500: Test #5 (Winter 2014)**

Solutions:

[12] 1. The domain of a function $f(x)$ is $(-\infty,\infty)$ and its first derivative is $f'(x)=(x-2)(x+1)$.
(a) What are the critical points of $f(x)$?
(b) Use the second derivative test to classify the local extrema of $f(x)$.
(c) Find the inflection points and the intervals where $f(x)$ is concave up, and where it is concave down.

Solution. (a) The critical points come from solving $f'(x)=0$, which in this case yields $x=2$ or $x=-1$.

(b) The second derivative is $f''(x)=2x-1$. Since $f''(2)>0$ it follows that we have a local minimum when $x=2$. Since $f''(-1)<0$ it follows that we have a local maximum when $x=-1$.

(c) $f''(x)>0$ when $x>\frac{1}{2}$, and $f''(x)<0$ when $x<\frac{1}{2}$. It follows that $f(x)$ is concave up over the interval $(\frac{1}{2},\infty)$, and it is concave down over the interval $(-\infty,\frac{1}{2})$. Consequently $x=\frac{1}{2}$ is the only inflection point.

[13] 2. The sum of two non-positive numbers is -20. What are these numbers if their product is maximal?

Solution.
Denote these numbers by x and y. So we have $x+y=-20$. We want to maximize their product $P=xy$. From $x+y=-20$ we find that $y=-20-x$. Plug in P to get $P=x(-20-x)=-20x-x^2$. We notice before proceeding that x must be in the interval $[-20,0]$. Now differentiate P: $P'=-20-2x$. Equating this to 0 gives $x=-10$ as the only critical point. The value of P when $x=-10$ is 100. Since $P(0)=P(-20)=0$, we have that he product is maximal when $x=-10$. Now compute $y=-20-(-10)=-10$.