MATH 1500: Test #4 (Winter 2014)

Solutions:

[8] 1. Find the inverse of the function $f(x) = \frac{x-1}{x+1}$. Your final answer should be a function on the variable x. Do NOT simplify your answer.

Solution. Start with $y = \frac{x-1}{x+1}$ and solve for x: $y(x+1) = x-1 \Leftrightarrow yx + y = x-1 \Leftrightarrow yx - x = -y-1$ which gives $x = \frac{y+1}{1-y}$. So the inverse function is $f^{-1}(x) = \frac{x+1}{1-x}$.

[8] 2. Find $\frac{dy}{dx}$ if $y = (2x)^{3x}$.

Solution. $\ln y = \ln(2x)^{3x} = 3x\ln(2x)$. So, after differentiating, $\frac{y'}{y} = 3\ln(2x) + 3x\frac{1}{2x}2$. This gives $y' = (2x)^{3x} \left(3\ln(2x) + 3x\frac{1}{2x}2 \right)$

[9] 3. Find the absolute extrema of the function $f(x) = x^3 - 12x$ over the interval [1,3].

Solution. Differentiate and solve f'(x) = 0; this means solving $3x^2 - 12 = 0$ yielding x = 2 or x = -2. The latter is out of our interval, so dump. Evaluate: f(1) = -11, f(2) = -16 and f(3) = -9. So, absolute maximum of -9 happens when x = 3, absolute minimum of -16 happens when x = 2.

B07.