B06. **MATH 1500: Test #3 (Winter 2014)**

Solutions:

[10] 1. Denote $f(x)=\dfrac{x+1}{x^2-1}$ and $g(x)=\sin(x^2)$

(a) Find $f'(x)$. Do NOT simplify your answer

(b) Find $\left(f(x)g[x]\right)'$. Do NOT simplify your answer

Solution.

(4) By the quotient rule, $f'(x)=\dfrac{(x+1)'\left(x^2-1\right)-(x+1)\left(x^2-1\right)'}{\left(x^2-1\right)^2}=\dfrac{\left(x^2-1\right)-(x+1)2x}{\left(x^2-1\right)^2}$.

(6) By the product rule

$$\left(f(x)g(x)\right)'=f'(x)g(x)+f(x)g'(x)=\frac{\left(x^2-1\right)-(x+1)2x}{\left(x^2-1\right)^2}\sin(x^2)+\frac{x+1}{x^2-1}\cos(x^2)2x$$

where in the last step we have used the chain rule.

[5] 2. Find $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\sin 2x}$.

Solution.

$$\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}=\lim_{x\to 0}\frac{\frac{\sin 4x}{4x}}{\frac{\sin 2x}{2x}}\left(\frac{4}{2}\right)=\frac{4}{2}=2\,.$$

[10] 3. The rim (edge) of a cube is increasing at the rate of $2\dfrac{km}{h}$. How fast is the volume of the cube changing at the moment when the rim is 3 km? Justify your answer.

Solution. If the volume of the cube is *V*, and the rim is *x*, then $V=x^3$. Differentiating with respect to time we get $\dfrac{dV}{dt}=\dfrac{dV}{dx}\dfrac{dx}{dt}=3x^2\cdot 2$. At the moment with $x=3$, we get $\dfrac{dV}{dt}=6\left(3^2\right)=54\dfrac{km}{h}$.