B05. **MATH 1500: Test #2 (Winter 2014)**

Solutions:

[16] 1. Denote $f(x)=\dfrac{x+1}{x^2-1}$.

(a) Find $\lim_{x\to -1} f(x)$.

(b) Find $\lim_{x\to \infty} f(x)$.

(c) What can you say about $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1^-} f(x)$?

(d) Are there any vertical or horizontal asymptotes of $f(x)$? Write down their equations, and briefly justify your answers.

Solution. (a) $\lim_{x\to -1} f(x)=\lim_{x\to -1}\dfrac{x+1}{x^2-1}=\lim_{x\to -1}\dfrac{x+1}{(x-1)(x+1)}=\lim_{x\to -1}\dfrac{1}{(x-1)}=-\dfrac{1}{2}$.

(b) .

$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\dfrac{x+1}{x^2-1}=\lim_{x\to\infty}\dfrac{x(1+\frac{1}{x})}{x(x-\frac{1}{x})}=\lim_{x\to\infty}\dfrac{(1+\frac{1}{x})}{(x-\frac{1}{x})}=0$, since the denominator tends to infinity while the numerator tends to 1.

(c) $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}\dfrac{x+1}{x^2-1}=\infty$, and $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\dfrac{x+1}{x^2-1}=-\infty$.

(d) It follows from part (c) that $x=1$ is a vertical asymptote, and it follows from part (b) that $y=0$ is a horizontal asymptote. ($y=0$ is also a horizontal asymptote to the left, but this is optional.)

[9] 2. (a) Write down the definition the derivative $f'(x)$ of a function $f(x)$.

(b) Use only the definition of the derivative as a limit to find $f'(x)$ if $f(x)=\sqrt{x+2}$. (No marks will be given if other methods are used.)

Solution.

(a) $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}=$$

(b) $$=\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\,\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}=\lim_{h\to 0}\frac{x+h+2-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})}=$$

$$=\lim_{h\to 0}\frac{1}{(\sqrt{x+h+2}+\sqrt{x+2})}=\frac{1}{2\sqrt{x+2}}$$