B04. **MATH 1500: Test #1 (Winter 2014)**

Solutions:

[8] 1. Find the domain of the function $\dfrac{\sqrt{x-2}}{(x-3)}$.

Solution. For the root to exist it must be $x-2 \geq 0$, that is x must be in $[2,\infty)$. The denominator should not be 0, which gives $x \neq 3$. So, the domain is $[2,3)$ together with $(3,\infty)$.

[9] 2. Which of the following functions is even, which is odd, which is neither even not odd?

$$f(x) = x2^{\left(x^2\right)}$$

$$g(x) = \frac{x^3+x}{x^{13}+x^3}$$

$$h(x) = x+1+\frac{1}{x^2}$$

Solution.

$f(-x) = (-x)2^{(-x)^2} = -x2^{\left(x^2\right)} = -f(x)$. Hence *f* is odd.

$g(-x) = \dfrac{(-x)^3+(-x)}{(-x)^{13}+(-x)^3} = \dfrac{-x^3-x}{-x^{13}-x^3} = \dfrac{x^3+x}{x^{13}+x^3} = g(x)$, and so g is even.

$h(-x) = -x+1+\dfrac{1}{(-x)^2} = -x+1+\dfrac{1}{x^2}$. This is neither $h(x)$ nor $-h(x)$. So h is neither even nor odd.

[8] 3. Evaluate the one-sided limits $\lim\limits_{x\to 1^+}\dfrac{x-1}{|x-1|}$ and $\lim\limits_{x\to 1^-}\dfrac{x-1}{|x-1|}$. Does $\lim\limits_{x\to 1^+}\dfrac{x-1}{|x-1|}$ exist? Justify briefly.

Solution.

$\lim\limits_{x\to 1^+}\dfrac{x-1}{|x-1|} = \lim\limits_{x\to 1^+}\dfrac{x-1}{x-1} = 1$;

$\lim\limits_{x\to 1^-}\dfrac{x-1}{|x-1|} = \lim\limits_{x\to 1^-}\dfrac{x-1}{-(x-1)} = -1$

Since these two limits are distinct it follows that $\lim\limits_{x\to 1^+}\dfrac{x-1}{|x-1|}$ does not exist.