[12] 1. Find each limit, if it exists. If the limit does not exist, indicate whether it is ∞ or $-\infty$, or neither .

(a) $\lim_{x\to 3}\frac{-x^2+2x+3}{x^3-3x^2}$

[4]

$$= \lim_{x\to 3} \frac{-(x-3)(x+1)}{x^2(x-3)}$$

$$= \lim_{x\to 3} \frac{-(x+1)}{x^2}$$

$$= -\frac{4}{9}$$

(b) $\lim_{x\to 0^-}\frac{8x+x^2}{|x|}$

[4]

$$= \lim_{x\to 0^-} \frac{x(8+x)}{-x}$$

$$= \lim_{x\to 0^-} -(8+x) = -8$$

(c) $\lim_{x\to -\infty}\frac{x-7}{\sqrt{x^2+x}}$

[4]

$$= \lim_{x\to -\infty} \frac{x-7}{\sqrt{x^2(1+1/x)}}$$

$$= \lim_{x\to -\infty} \frac{x-7}{-x\sqrt{1+1/x}}$$

$$= \lim_{x\to -\infty} \frac{1-7/x}{-\sqrt{1+1/x}}$$

$$= \frac{1}{-\sqrt{1}} = -1$$

Values

[6] 2. Find the value(s) of k so that the function

$$f(x)=\begin{cases}\dfrac{(x+1)(x-k)}{x+1} & \text{if } x<-1\\ 5-2xk & \text{if } x\geq -1\end{cases}$$

is continuous at $x=-1$. Use limits to justify your answers

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} \frac{(x+1)(x-k)}{x+1} = \lim_{x\to -1^-}(x-k) = -1-k$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 5-2xk = 5+2k\cdot 1$$

For $\lim\limits_{x\to -1} f(x)$ to exist $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^-} f(x)$

$$\therefore\quad 5+2k=-1-k$$

$$3k=-6$$

$$k=-2$$

when $k=-2$, $\lim\limits_{x\to -1} f(x) = 1 = f(-1)$

Making f continuous at $x=-1$

Values

[15] 3. Find $f'(x)$. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $f(x)=\frac{5}{\sqrt[3]{x^2}}+x^e+e^x+e^\pi$

$f(x)=5x^{-2/3}+x^e+e^x+e^\pi$

$f'(x)=-\frac{10}{3}x^{-5/3}+ex^{e-1}+e^x$

[4]

(b) $f(x)=(\tan x-\sec x)x^7$

$f'(x)=(\sec^2 x-\sec x\tan x)x^7+(\tan x-\sec x)(7x^6)$

[4]

$f(x)=x^7\tan x-x^7\sec x$

$\therefore f'(x)=7x^6\tan x+x^7\sec^2 x-7x^6\sec x-x^7\sec x\tan x$

(c) $f(x)=\frac{1+\cos x}{1-\sin x}$

$f'(x)=\frac{-\sin x(1-\sin x)-(1+\cos x)(-\cos x)}{(1-\sin x)^2}$

[4]

$f(x)=(1+\cos x)(1-\sin x)^{-1}$

$f'(x)=-\sin x(1-\sin x)^{-1}+(1+\cos x)(-1)(1-\sin x)^{-2}(-\cos x)$

(d) $f(x)=e^{3x^5-x^2+9}$

[3] $f'(x)=e^{3x^5-x^2+9}(15x^4-2x)$

Values

[6] 4. Using the definition of the derivative find $f'(x)$ if $f(x)=\sqrt{x-3}$.

No marks will be given if other methods are used.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h}\cdot\frac{\sqrt{x+h-3}+\sqrt{x-3}}{\sqrt{x+h-3}+\sqrt{x-3}}$$

$$=\lim_{h\to 0}\frac{x+h-3-(x-3)}{h\left[\sqrt{x+h-3}+\sqrt{x-3}\right]}$$

$$=\lim_{h\to 0}\frac{h}{h\left[\sqrt{x+h-3}+\sqrt{x-3}\right]}$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h-3}+\sqrt{x-3}}$$

$$=\frac{1}{\sqrt{x-3}+\sqrt{x-3}}=\frac{1}{2\sqrt{x-3}}$$

[7] 5. Find an equation of the tangent line to the curve described by $x^3+xy+y^3=x-6$ at the point with coordinates $(2,-2)$.

$$3x^2 + y + x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 1$$

at $(2,-2)$

$$3(2)^2 + (-2) + 2m + 3(-2)^2 m = 1$$

$$12 - 2 + 2m + 12m = 1$$

$$14m = -9$$

$$m = -\frac{9}{14}$$

Equation of tangent line at $(2,-2)$ is:

$$y + 2 = -\frac{9}{14}(x-2)$$

[4] 6. Prove the following theorem:

If f and g are differentiable functions then

$$(f+g)'(x) = f'(x) + g'(x)$$

$$(f+g)'(x) = \lim_{h\to 0}\frac{(f+g)(x+h) - (f+g)(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} + \lim_{h\to 0}\frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

Values

[3] 7. The position of a particle moving along the x-axis is given by $x(t) = t^4 - 3t^3 - 8t^2 + 7$ m. If t is in seconds, find the acceleration when $t = 2$ s.

$$v(t) = x'(t) = 4t^3 - 9t^2 - 16t$$

$$a(t) = v'(t) \text{ or } x''(t) = 12t^2 - 18t - 16$$

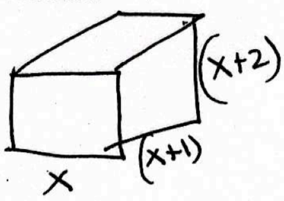
at $t = 2$s

$$a(2) = 12(2)^2 - 18(2) - 16 = 48 - 36 - 16$$

$$= -4 \text{ m/s}^2$$

[7] 8. A rectangular solid (a box) has a length of x metres, a width of $(x+1)$ metres, and a height of $(x+2)$ metres. Therefore, its volume is given by $V = x^3 + 3x^2 + 2x$ and its surface area is given by $A = 6x^2 + 12x + 4$. When $x = 2$ metres the volume is increasing at $78\ m^3/s$. At what rate is the surface area changing at this instant?

$$\frac{dV}{dt} = 78 \text{ m}^3/\text{s} \qquad \frac{dA}{dt} = ?$$



$$V = x^3 + 3x^2 + 2x$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt} + 6x\frac{dx}{dt} + 2\frac{dx}{dt} \text{ or } (3x^2 + 6x + 2)\frac{dx}{dt}$$

When $x = 2$m

$$78 = \boxed{3(2)^2 + 6(2) + 2}\frac{dx}{dt}$$

$$78 = (12 + 12 + 2)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{78}{26} = 3 \text{ m/s}$$

$$A = 6x^2 + 12x + 4$$

$$\frac{dA}{dt} = 12x\frac{dx}{dt} + 12\frac{dx}{dt} \text{ or } (12x + 12)\frac{dx}{dt}$$

when $x = 2$m

$$\frac{dA}{dt} = (12(2) + 12)3 = (36)(3) = 108$$

the ~~su~~surface area is increasing at 108 m^2/s.