

B18.

**MATH 1500: Test #3 (Fall 2010)****Solution; marking scheme**

Always give (brief) justification, and show your work.

[15] 1. Find  $f'(x)$ :

$$[5] \text{ (a) } f(x) = [\sin(x^2)](\cos^2 x)$$

$$[5] \text{ (b) } f(x) = \frac{\sec x}{e^{2x}}$$

$$[5] \text{ (c) } f(x) = 3^{\sqrt{1+\sqrt{x}}}$$

*Solution.* (a)  $f'(x) = [2x \cos(x^2)](\cos^2 x) + [\sin(x^2)][2 \cos x(-\sin x)]$

$$\text{(b) } f'(x) = \frac{(\sec x)(\tan x)e^{2x} - (\sec x)e^{2x} 2}{e^{4x}}$$

$$\text{(c) } f'(x) = 3^{\sqrt{1+\sqrt{x}}} (\ln 3) \frac{1}{2\sqrt{1+\sqrt{x}}} \frac{1}{2\sqrt{x}}$$

[10] 2. Find an equation of the tangent line to the curve  $y^3 \sin x + 2y + x = 2$  at the point  $(0,1)$ .

*Solution.* Differentiate implicitly:  $3y^2 y' \sin x + y^3 \cos x + 2y' + 1 = 0$ . Substitute  $x = 0$  and  $y = 1$ , to get  $1 + 2y' + 1 = 0$ . From there we find that  $y' = -1$ . So  $\frac{y-1}{x} = -1$  is an equation of the tangent line.