B17. **MATH 1500: Test #2 (Fall 2010)**

Solution; marking scheme

Always give (brief) justification, and show your work.

[5] 1. Find all vertical and horizontal asymptotes of the function $f(x)=\dfrac{1}{x-2}+1$.

Solution. VA: $\lim_{x\to 2^+}\dfrac{1}{x-2}+1=\infty$ (and $\lim_{x\to 2^-}\dfrac{1}{x-2}+1=-\infty$), and so $x=2$ is a VA.
HA: $\lim_{x\to\infty}\dfrac{1}{x-2}+1=1$ (and $\lim_{x\to-\infty}\dfrac{1}{x-2}+1=1$), so $y=1$ is a HA.

[10] 2. Use **only** the definition of derivative to find $f'(0)$ if $f(x)=\sqrt{x+1}$.

Solution.
$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}=\lim_{h\to 0}\frac{\sqrt{h+1}-1}{h}\,\frac{\sqrt{h+1}+1}{\sqrt{h+1}+1}=$$
$$=\lim_{h\to 0}\frac{h+1-1}{h(\sqrt{h+1}+1)}=\lim_{h\to 0}\frac{1}{\sqrt{h+1}+1)}=\frac{1}{2}.$$

[10] 3. Find an equation of the tangent line to the curve $f(x)=x^3+3x-2$ at the point corresponding to $x=1$. (You may use the basic properties of differentiation in this question.)

Solution. $f'(x)=3x^2+3$, and so $f'(1)=6$. This is the slope of the tangent line at the given point. [Note: from here on there are many ways to complete the solution; the following is just one of them.) So, its equation is $y=6x+b$. The point where this line touches the curve is $(1,f(1))=(1,2)$; so $2=(6)(1)+b$, from where we find $b=-4$. So, $y=6x-4$ is an equation of the tangent line.