$-\infty$, or neither. Justify your answers.

3 **(a)** $\lim_{x\to 3} \frac{x^2-x-6}{x-3}$ $= \lim_{x\to3} \frac{(x-3)(x+2)}{x-3} = \lim_{x\to3} (x+2) = 3+2 = 5$

2 1 1

2 **(b)** $\lim_{x\to 3} \frac{x^2-x-6}{x+3}$ $= \frac{9-3-6}{6} = 0$ (Because: a rational function is continuous where defined, OR: "By the Direct Substitution Property")

2 1

2.

5 **(c)** $\lim_{x\to -1^-} \frac{1-x^2}{|1+x|}$ .. SINCE $x < -1$, then $1+x<0$, so $|1+x| = -(1+x)$.

1 2

Then $\lim_{x\to-1^-} \frac{1-x^2}{|1+x|} = \lim_{x\to-1^-} \frac{(1-x)(1+x)}{-(1+x)} = -\lim_{x\to-1^-} (1-x) = -(1-(-1))$

$= -2$

1 1

2

6 **(d)** $\lim_{x\to -\infty} \frac{2x+3}{\sqrt{7x+3x^2}}$ $= \lim_{x\to-\infty} \frac{2x+3}{\sqrt{x^2(\frac{7}{x}+3)}} = \lim_{x\to-\infty} \frac{2x+3}{\sqrt{x^2}} \cdot \frac{1}{\sqrt{\frac{7}{x}+3}}$

1

If $x<0$, then $\sqrt{x^2} = -x$, so the limit is

1

$L = \lim_{x\to-\infty} \frac{2x+3}{-x} \cdot \frac{1}{\sqrt{7/x+3}} = \lim_{x\to-\infty} \left(-2-\frac{3}{x}\right) \cdot \frac{1}{\sqrt{7/x+3}}$.

1

Since $\frac{3}{x} \to 0$ and $\frac{7}{x}\to 0$ as $x\to-\infty$, $L = (-2-0)\cdot\frac{1}{\sqrt{0+3}} = \frac{-2}{\sqrt{3}}$

1 1

[Values]

[16] **2.** Find the derivative of each function. DO NOT SIMPLIFY.

5 **(a)** $y = \dfrac{9}{\sqrt[3]{x^2}} - x^e + \pi^2 + \cot(x^2)$

$= 9x^{-2/3} - x^e + \pi^2 + \cot(x^2)$

$\therefore y' = 9(-\frac{2}{3})x^{-5/3} - ex^{e-1} + 0 - \csc^2(x^2)\cdot 2x$

1 1 1 1 1

4 **(b)** $y = (\sec x + e^{3x})(x^3 + x^{-3})$

$y' = (\sec x + e^{3x})'(x^3 + x^{-3}) + (\sec x + e^{3x})(x^3 + x^{-3})'$ 1 for product rule

$= (\sec x \tan x + e^{3x}\cdot 3)(x^3 + x^{-3}) + (\sec x + e^{3x})(3x^2 - 3x^{-4})$

1 1 1

4 **(c)** $f(x) = \dfrac{x + \cos x}{2 + \sin x}$.

$f'(x) = \dfrac{(2+\sin x)(x+\cos x)' - (2+\sin x)'(x+\cos x)}{(2+\sin x)^2}$ 2 for quotient rule

$= \dfrac{(2+\sin x)(1-\sin x) - (0+\cos x)(x+\cos x)}{(2+\sin x)^2}$

1 1

3 **(d)** $g(t) = e^{\sqrt{\sin t}}$

$g'(t) = e^{\sqrt{\sin t}}\cdot(\sqrt{\sin t})'$ 1

$= e^{\sqrt{\sin t}}\cdot\dfrac{1}{2\sqrt{\sin t}}\cdot(\sin t)'$ 1

0 FOR NO CHAIN RULE

$= e^{\sqrt{\sin t}}\cdot\dfrac{1}{2\sqrt{\sin t}}\cos t$ 1

[Values]

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1 1 1 1 1

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1 1 1

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1 1

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$$g'(t) = e^{\sqrt{\sin t}}\cdot(\sqrt{\sin t})'$$ 1

$$= e^{\sqrt{\sin t}}\cdot\frac{1}{2\sqrt{\sin t}}\cdot(\sin t)'$$ 1

0 FOR NO CHAIN RULE

$$= e^{\sqrt{\sin t}}\cdot\frac{1}{2\sqrt{\sin t}}\cos t$$ 1

[Values]

[6] **5.** Prove that if a function $f(x)$ is differentiable at a number a, then $f(x)$ is continuous at a.

?

Let $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exist. Let $Q(x) = \dfrac{f(x) - f(a)}{x - a}$.

Then $\lim_{x \to a} Q(x) \cdot (x - a) \overset{1}{=} \lim_{x \to a} Q(x) \cdot \lim_{x \to a} (x - a) = f'(a) \cdot 0 = 0$ 1

So $\lim_{x \to a} f(x) - f(a) = 0$. 1 Then $\lim_{x \to a} f(x) = \lim_{x \to a} \left[f(x) - f(a) + f(a) \right]$

1

$= \lim_{x \to a} f(x) - f(a) + \lim_{x \to a} f(a) = 0 + f(a) = f(a)$.

2 $\boxed{\text{Since } \lim_{x \to a} f(x) = f(a), \ f \text{ is continuous at } a.}$

[7] **6.** Find an equation of the tangent line to the curve

$$e^y + 2xy - 2x^2 = -1 \qquad (*)$$

at the point $(-1, 0)$.

Differentiating $(*)$ w.r.t. x:

3 $e^y y' + 2y + 2xy' - 2 \cdot 2x = 0$

$\therefore$ 1 $y'(e^y + 2x) = 4x - 2y$

Putting $x = -1$ and $y = 0$: $\quad y'(e^0 - 2) = -4 - 0$

$\therefore y' = \dfrac{-4}{-\cancel{2}\,1} = \cancel{2}\ 4$ 1

Tangent line is $\boxed{y - y_0 = m(x - x_0)}$, where $x_0 = -1$, $y_0 = 0$ and $m = \cancel{2}\ 4$.

1

$y - 0 = \cancel{2}\,4(x+1)$ or $\boxed{y = \cancel{2}\,4x + \cancel{2}\,4}$

1