B4.

MATH 1500: Test #5 (Winter 2009)
Solutions

[17] 1. Consider the function $f(x)=\dfrac{x^2}{x-1}$. The first and the second derivatives are $f'(x)=\dfrac{x^2-2x}{(x-1)^2}$ and $f''(x)=\dfrac{2}{(x-1)^3}$ (no need to check this).

[9] **(a)** Where is $f(x)$ increasing, where is it decreasing?

[7] **(b)** Where is $f(x)$ concave up, where is it concave down?

Solution. ***(a)*** Critical points are $x=2$ and $x=0$ (we get them by solving $f'(x)=0$). These split the domain into $(-\infty,0)$, $(0,1)$, $(1,2)$ and $(2,\infty)$. Choosing numbers in each of these and evaluating the sigh of $f'(x)$, we see the following:

Over $(-\infty,0)$, $f'(x)$ is positive; so over that interval $f(x)$ increases.
Over $(0,1)$, $f'(x)$ is negative; so over that interval $f(x)$ decreases.
Over $(1,2)$, $f'(x)$ is negative; so over that interval $f(x)$ decreases.
Over $(2,\infty)$, $f'(x)$ is positive; so over that interval $f(x)$ increases

(b) It is obvious that $f''(x)$ is negative for negative $(x-1)$ and positive for positive $(x-1)$. So,

$f(x)$ is concave down for $x-1<0$, i.e., for $x<1$
$f(x)$ is concave up for $x-1>0$, i.e., for $x>1$

[9] 2. Suppose the perimeter of a rectangle is 12. What are the dimensions of such a rectangle with maximal area? Justify your answer using derivatives.

Solution. Denote the lengths of the sides of the rectangle by x and y. Then we are given that $2x+2y=12$, that is, $x+y=6$. The area of the rectangle is $A(x)=xy$. It follows from $x+y=6$ that $y=6-x$. Substituting this in the formula for area we get $A(x)=x(6-x)=6x-x^2$. We want to maximize this function. Compute: $A'(x)=6-2x$ and solving $A'(x)=0$ gives $x=3$. This is the only C.P. Since $A''(x)=-2<0$ the critical point gives a local maximum. Since this is the only local extremum (and since the function $A(x)$ is differentiable; no points deducted if that is not mentioned) it follows that this local maximum is absolute maximum. The other dimension can be computed from $y=6-x$, so that $y=3$ too.