B7.

MATH 1500: Test #4 (Winter 2009)
Solutions

[8] 1. The base of a rectangle is fixed at 5 meters, while the height is increasing at the rate of 4 meters per second. How fast is the area of the rectangle increasing at the time when the height is 100 meters.

Solution. Denote the area by A, the length of the base by y and the height by x. Then $A = xy$, and since $y = 5$, we have that $A = 5x$. The rate of change of A with respect to time is $\frac{dA}{dt} = 5\frac{dx}{dt} = 5(4) = 20 \,{}^{m}\!/_{\text{sec}}$.

[9] 2. Find $\frac{dy}{dx}$ if $y = (\sin x)^x$.

Solution. We have $\ln y = \ln(\sin x)^x = x\ln(\sin x)$. Differentiate implicitly to get:

$\frac{1}{y}y' = \ln(\sin x) + x\frac{1}{\sin x}\cos x$. So

$y' = y\,(\ln(\sin x) + x\frac{1}{\sin x}\cos x) = (\sin x)^x(\ln(\sin x) + x\frac{1}{\sin x}\cos x)$.

[8] 3. Find the critical points (critical numbers) of the functions $f(x) = x^3 - 12x$.

Solution. $f'(x) = 3x^2 - 12$ and it always exists. So the CP-s come from solving $3x^2 - 12 = 0$ which gives $x = 2$ or $x = -2$.