B6.

MATH 1500: Test #3 (Winter 2009)
Solutions

[15] 1. Find $f'(x)$ for the following functions (do not simplify):

(a) $f(x) = \sqrt[5]{x} + e\pi$

(b) $f(x) = (\cos x)(\sin x)$

(c) $f(x) = \cos(\sin x)$

(d) $f(x) = \dfrac{x}{2x + x^3}$

(e) $f(x) = \sqrt{e^{\sec x}}$

Solution.

(a) $f'(x) = \dfrac{1}{5}x^{-\frac{4}{5}}$

(b) $f'(x) = -(\sin x)(\sin x) + (\cos x)\cos x$

(c) $f'(x) = [-\sin(\sin x)]\cos x$

(d) $f'(x) = \dfrac{(2x + x^3) - x(2 + 3x^2)}{(2x + x^3)^2}$

(e) $f'(x) = \dfrac{1}{2\sqrt{e^{\sec x}}}e^{\sec x}\tan x \sec x$ (or $f'(x) = \dfrac{1}{2\sqrt{e^{\sec x}}}e^{\sec x}\dfrac{\sin x}{\cos^2 x}$)

[10] 2. Find the equation of the tangent line to the curve defined by $xy^2 + x^2y^5 = 6$ at the point $(2,1)$.

Solution.

The equation is $y - 1 = m(x - 2)$, where $m = \dfrac{dy}{dx}$ at the point $(2,1)$.

Differentiate implicitly: $y^2 + x2y\dfrac{dy}{dx} + 2xy^5 + x^2 5y^4 \dfrac{dy}{dx} = 0$. Put $x = 2,\ y = 1$ to get $1 + 4\dfrac{dy}{dx} + 4 + 20\dfrac{dy}{dx} = 0$, from where we find $\dfrac{dy}{dx} = -\dfrac{5}{24}$. So the equation of the tangent line is $y - 1 = -\dfrac{5}{24}(x - 2)$.