B5.

MATH 1500: Test #2 (Winter 2009)
Solutions

1. Evaluate $\lim_{x\to\infty}(x-\sqrt{x^2-1})$. Write the equation of one horizontal asymptote for the function $f(x)=x-\sqrt{x^2-1}$.

Solution.

(a) $\lim_{x\to\infty}(x-\sqrt{x^2-1})=\lim_{x\to\infty}(x-\sqrt{x^2-1})\dfrac{(x+\sqrt{x^2-1})}{(x+\sqrt{x^2-1})}=\lim_{x\to\infty}\dfrac{(x^2-x^2+1)}{(x+\sqrt{x^2-1})}=0$.

Asymptote: $y=0$.

2. Is the function $f(x)=\begin{cases}\dfrac{x-1}{x^2-3x+2} & \text{if } x>1\\ -1 & \text{if } x\le 1\end{cases}$ continuous at $x=1$? Justify your answer.

Solution. $f(1)=-1$.

$\lim_{x\to1^+}f(x)=\lim_{x\to1^+}\dfrac{x-1}{x^2-3x+2}=\lim_{x\to1^+}\dfrac{x-1}{(x-1)(x-2)}=\lim_{x\to1^+}\dfrac{1}{(x-2)}=-1$.

$\lim_{x\to1^-}f(x)=\lim_{x\to1^-}(-1)=-1$.

Since all of these are equal, the function is continuous at $x=1$.

3. Use the definition of derivative only to find $f'(2)$ if $f(x)=3x+1$. No points will be given if other methods are used.

Solution. $f'(2)=\lim_{h\to0}\dfrac{f(2+h)-f(2)}{h}=\lim_{h\to0}\dfrac{3(2+h)+1-(3(2)+1)}{h}=\lim_{h\to0}\dfrac{3h}{h}=3$.