Values:

1. Find the limit or explain why the limit does not exist.

[2] a) $\lim_{x\to 2^+} \frac{2-x}{|2-x|}$,

$$\lim_{x\to 2^+} \frac{2-x}{|2-x|} = \lim_{x\to 2^+} \frac{2-x}{x-2} = \lim_{x\to 2^+} (-1) = -1.$$

[2] b) $\lim_{x\to -3} \frac{x+3}{3+\sqrt{3-2x}}$, $= \frac{-3+3}{3+\sqrt{3-2(-3)}} = 0$

[3] c) $\lim_{x\to 0} \frac{\sin 2x}{3x}$, $= \lim_{x\to 0} \frac{2}{3}\frac{\sin 2x}{2x} = \frac{2}{3}\lim_{x\to 0}\frac{\sin 2x}{2x}$

$$= \frac{2}{3}\cdot 1 = \frac{2}{3}$$

iT

[3] d) $\lim_{x\to 3} \frac{2-\sqrt{x+1}}{3-x} = \lim_{x\to 3} \frac{(2-\sqrt{x+1})(2+\sqrt{x+1})}{(3-x)(2+\sqrt{x+1})}$

$$= \lim_{x\to 3} \frac{4-(x+1)}{(3-x)(2+\sqrt{x+1})} = \lim_{x\to 3} \frac{\cancel{3-x}}{(\cancel{3-x})(2+\sqrt{x+1})}$$

$$= \lim_{x\to 3} \frac{1}{2+\sqrt{x+1}} = \frac{1}{2+\sqrt{4}} = \frac{1}{4}.$$

Values:

[7] 2. Find the value or values of k such that the function

$$f(x)=\begin{cases} k^2x^2+kx & x<3, \\ 6 & x=3, \\ x^2-k^2x & x>3, \end{cases}$$

is continuous at $x=3$.

$\lim_{x\to 3^-}(k^2x^2+kx) = 9k^2+3k = 6$ for continuity (1)

$\lim_{x\to 3^+}(x^2-k^2x) = 9-3k^2 = 6$ for continuity (2)

From (1) $3k^2+k-2=0$

$(3k-2)(k+1)=0$

$3k-2=0$ or $k+1=0$

$k=\frac{2}{3}$ or $k=-1$

From (2) $3k^2=3$, $k^2=1$, $k=\pm 1$.

(Common value and only possibility is $k=-1$.

Values:

3. Find $\frac{dy}{dx}$. Do <u>not</u> simplify your answer.

[3] a) $y = \sin(\cos x)$,

$$y' = \cos(\cos x)(-\sin x)$$

[3] b) $y = \sqrt[4]{x^9} + \left(\frac{3}{2}\right)^2 - e^{x^2}$, $= x^{9/4} + \left(\frac{3}{2}\right)^2 - e^{x^2}$

$$y' = \frac{9}{4} x^{5/4} - e^{x^2} \cdot 2x$$

[3] c) $y = \frac{\cos x}{1+\sqrt{x}}$,

$$y' = \frac{(1+\sqrt{x})(-\sin x) - \cos x\left(\frac{1}{2}x^{-1/2}\right)}{(1+\sqrt{x})^2}$$

~~DUC~~

XUCF.

[3] d) $y = (\sin x)\sqrt{\pi - x}$.

$$y' = (\sin x)\left(\frac{1}{2\sqrt{\pi - x}}(-1)\right) + (\cos x)\sqrt{\pi - x}$$

NCF.

Values:

[3] **4.** **a)** **When is a function $f(x)$ differentiable at $x = a$? (State the definition.)** The function $f(x)$ is differentiable at $x = a$ if $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ exists.

[6] **b)** **Use <u>only</u> the definition of the derivative (part (a) of this question) to compute $f'(a)$ if $f(x) = x^2 - 2x$.**

$$\lim_{h \to 0} \frac{(a+h)^2 - 2(a+h) - a^2 + 2a}{h}$$

$$= \lim_{h \to 0} \frac{a^2 + 2ah + h^2 - 2a - 2h - a^2 + 2a}{h}$$

$$= \lim_{h \to 0} \frac{2ah - 2h + h^2}{h}$$

$$= \lim_{h \to 0} 2a - 2 + h$$

$$= 2a - 2.$$

[6] **5.** **Suppose $f(x)$ and $g(x)$ are differentiable functions. Prove that $(f(x) + g(x))' = f'(x) + g'(x)$.**

$$(f(x) + g(x))' = \lim_{h \to 0} \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

provided ~~that~~ $f'(x)$ and $g'(x)$ exist.

THE UNIVERSITY OF MANITOBA

DATE: February 23, 2007 — MIDTERM EXAMINATION

DEPARTMENT & COURSE NO: MATH 1500 —

EXAMINATION: Introductory Calculus — TIME: 1 hour

EXAMINER: Various

Values:

6. a) The equation $y^3 = \dfrac{4x-2y}{x+y}$ defines y implicitly as a function of x.

[6] Find the value of the derivative y' at the point $(1,1)$.

$xy^3 + y^4 = 4x - 2y$ provided $x \neq -y$.

$x\,3y^2y' + y^3 + 4y^3y' = 4 - 2y'$

Let $x = y = 1$. $3y' + 1 + 4y' = 4 - 2y'$

$9y' = 3$

$y' = \frac{3}{9} = \frac{1}{3}$

OR $(3xy^2 + 4y^3 + 2)y' = 4 - y^3$

$$y' = \frac{4-y^3}{3xy^2+4y^3+2} = \frac{4-1}{3+4+2} \quad \text{if } x=y=1.$$

$y' = \frac{1}{3}$

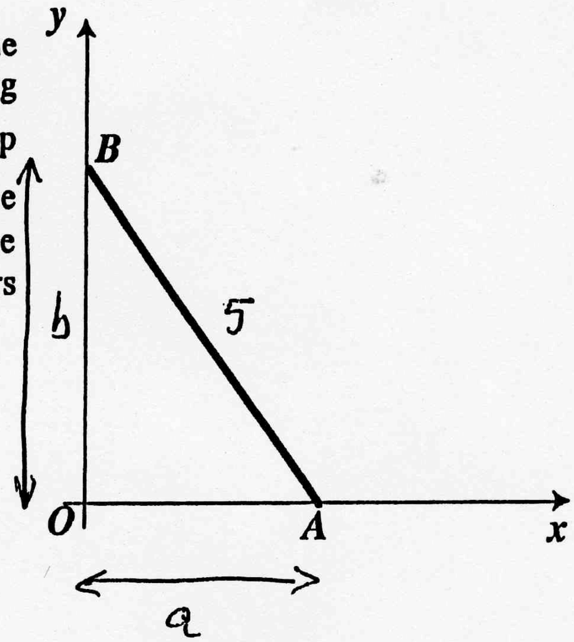
b) Find the equation of the tangent line to the curve determined by

[2] $y^3 = \dfrac{4x-2y}{x+y}$ at the point $(1,1)$.

$y - 1 = \frac{1}{3}(x-1).$

Values:

[8] 7. The line segment AB is 5 meters long. The bottom A slides away from the origin O along the x-axis at the rate of $2\,{}^{m}\!/_{\text{sec}}$, while the top B slides down along the y-axis (see the illustration). How fast does B approach the origin O at the moment when A is 3 meters from O?



$$a^2 + b^2 = 5^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

If $a = 3$ and $b = 4$ and $\frac{da}{dt} = 2$, then

$$\cancel{2}\cdot 3\cdot\cancel{2} = -\cancel{2}\cdot\overset{2}{\cancel{4}}\cdot\frac{db}{dt}$$

$$\frac{db}{dt} = -\frac{3}{2}\ \text{m/sec.}$$

Therefore B approaches O at a rate of $\frac{3}{2}$ m/sec.