B06.

# Math 1500: Quiz #6
# Solutions

**Solutions**

**[8]** **1.** Find the second derivative $f''(x)$.

(a) $f(x) = \cos(x^2)$

(b) $f(x) = xe^{2x}$

**Solution**. (a) $f'(x) = -2x\sin(x^2)$ and $f''(x) = -2\sin(x^2) - (2x)(2x)\cos(x^2)$

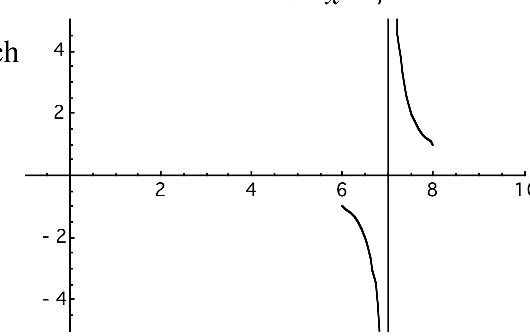
(b) $f'(x) = e^{2x} + 2xe^{2x}$ and $f''(x) = 2e^{2x} + 2e^{2x} + 2x(2e^{2x})$.

**[8]** **2.** **(a)** Evaluate $\lim\limits_{x\to\infty} \dfrac{3x^2 + 2x}{\sqrt{x^4 - x}}$.

**(b)** Find all the vertical asymptotes of the function $f(x) = \dfrac{1}{x-7}$ and sketch the graph of this function around its vertical asymptotes. (Note again that you only need to sketch the graph around the vertical asymptote, not all of the graph.)

***Solution***. **(a)** $\lim\limits_{x\to\infty} \dfrac{3x^2+2x}{\sqrt{x^4-x}} = \lim\limits_{x\to\infty} \dfrac{3x^2+2x}{\sqrt{x^4(1-\frac{1}{x^3})}} = \lim\limits_{x\to\infty} \dfrac{x^2(3+\frac{2}{x})}{x^2\sqrt{(1-\frac{1}{x^3})}} = \lim\limits_{x\to\infty} \dfrac{(3+\frac{2}{x})}{\sqrt{(1-\frac{1}{x^3})}} = 3$

**(b)** The only potential vertical asymptote is $x = 7$. We check: $\lim\limits_{x\to 7^+} \dfrac{1}{x-7} = \infty$ and $\lim\limits_{x\to 7^-} \dfrac{1}{x-7} = -\infty$. So, $x = 7$ is a vertical asymptote. Sketch shown to the right.




**[9]** **3.** Find the inverse $f^{-1}(x)$ of the function $f(x) = \dfrac{x-3}{x-2}$. Show your work!

***Solution.*** Set $y = \dfrac{x-3}{x-2}$ and solve for *x* in terms of *y*:

$y(x-2) = x-3 \Leftrightarrow xy - 2y = x - 3 \Leftrightarrow xy - x = 2y - 3 \Leftrightarrow x(y-1) = 2y-3 \Leftrightarrow x = \dfrac{2y-3}{y-1}$.

So, $f^{-1}(x) = \dfrac{2x-3}{x-1}$.