B05.

Math 1500: Quiz #2
20 minutes

Solutions

[8] **1.** Given $f(x)=\begin{cases} cx+2 & \text{if } x\le 1 \\ x+2c & \text{if } x>1 \end{cases}$ compute $f(1)$, $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1^-} f(x)$. Find the constant c such that the function $f(x)$ is continuous at $x=1$.

Solution. $f(1)=c+2$, $\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(x+2c)=1+2c$, $\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(cx+2)=c+2$. The function is continuous at $x=1$ if all of these are equal, which gives $1+2c=c+1$. Solving this yields $c=0$.

[8] **2.** **(a)** Find $f'(x)$ if $f(x)=xe^x$.

(b) Find an equation of the tangent line to the curve $f(x)=xe^x$ at the point when $x=0$.

***Solution*.** **(a)** By the product rule we have $f'(x)=e^x+xe^x$.

(b) The slope of the tangent line to the curve $f(x)=xe^x$ at the point when $x=0$ is $f'(0)=1$. Since $f(0)=0$ the tangent line passes through the point $(0,0)$. So, its equation is $\frac{y}{x}=1$ or $y=x$.

[9] **3.** **(a)** Compute $\lim_{x\to 0}\frac{x}{\sin(2007x)}$

(b) Differentiate $f(x)=\frac{e^x\tan x}{2\cos x}$. Do not simplify!

Solution. **(a)** $\lim_{x\to 0}\frac{x}{\sin(2007x)}=\lim_{x\to 0}\frac{2007x}{2007\sin(2007x)}=\frac{1}{2007}\lim_{x\to 0}\frac{2007x}{\sin(2007x)}=\frac{1}{2007}$.

(b)

$$f'(x)=\frac{\left(e^x\tan x\right)'(2\cos x)-\left(e^x\tan x\right)(2\cos x)'}{4\cos^2 x}=\frac{\left(e^x\tan x+e^x\frac{1}{\cos^2 x}\right)(2\cos x)-\left(e^x\tan x\right)(-2\sin x)}{4\cos^2 x}$$