B04.

Math 1500: Quiz #1
20 minutes
Solutions and marking scheme

[5] **1.** Find the domain of the function $f(x)=\dfrac{\sqrt{x-2}}{x-7}$ and express it in terms of intervals.

Solution. $x-2\geq 0$ because of the root in the numerator. This tells us that $x\geq 2$.
$x-7\neq 0$ because of the denominator. So, $x\neq 7$.
So, the domain of this function are the intervals $[2,7)$ and $(7,\infty)$.

[10] **2.** Is the function $f(x)$ even , odd or neither? Do not forget to show justification.

(a) $f(x)=\dfrac{x^3}{x^5-x}$

(b) $f(x)=\dfrac{x^3}{x^5-1}$

Solution. **(a)** $f(-x)=\dfrac{(-x)^3}{(-x)^5-(-x)}=\dfrac{-x^3}{-x^5+x}=\dfrac{x^3}{x^5-x}=f(x)$ and so this function is even.

(b) $f(-x)=\dfrac{(-x)^3}{(-x)^5-1}=\dfrac{-x^3}{-x^5-1}$ and since this is neither $f(x)$ nor $-f(x)$ this function is neither even nor odd.

[10] **3.** Compute each of the following limits, or show it does not exist.

(a) $\displaystyle\lim_{x\to -2}\frac{2+x}{8-x^3}$

(b) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

Solution. **(a)** Since –2 is in the domain of the rational function $\dfrac{2+x}{8-x^3}$ we simply substitute $x=-2$ in $\dfrac{2+x}{8-x^3}$: $\displaystyle\lim_{x\to -2}\frac{2+x}{8-x^3}=\frac{0}{8-(-8)}=0$.

(b) $\displaystyle\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}\frac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{1}{(\sqrt{x}+1)}=\frac{1}{2}$.

[Alternatively: $\displaystyle\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}=\lim_{x\to 1}\frac{1}{(\sqrt{x}+1)}=\frac{1}{2}$ where in the first step we have used the difference of squares identity.]