Mini Exam (bonus)

1 Suppose $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{a_n}$ for n=1,2,.... Is the sequence $\{a_n\}$ convergent ? Justify your answer in **not more than two sentences**.

Bounded & decreasing

2 The sum of the series $\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}}$ is

A. ∞ B. $\frac{2}{3}$ C. 1 D. $\frac{3}{2}$ E. None of these.

3 The Ratio Test applied to $\sum_{n=1}^{\infty} \frac{1}{n}$

A. Yields that series diverges. B. Yields that series converges. C. Is inconclusive,

4 The integral test can be applied to:

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ B. $\sum_{n=0}^{\infty} \frac{3^n}{2^n}$ C. $\sum_{n=0}^{\infty} \frac{1}{2^n}$ D. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

5 The series $\sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^n}$

A. Converges absolutely. B. Converges conditionally. C. Diverges.

6 The center of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n (3x-2)^n}{2^n}$ is

A. $\frac{2}{3}$ B. 0 C. 2 D. None of these.

7. The radius of convergence of $\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ is

A. 0 B. 2. C. 4 D. $\frac{1}{2}$.

8. The interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is

A. 0 B. $(-1,1)$ C. ∞ D. $(-\infty,\infty)$ E. None of these.

9. The sum of $\sum_{n=2}^{\infty} \frac{x^n}{n!}$ is

A. $e^x - e - 1$ B. $e^x - e$ C. e^x D. $\frac{1}{1-x} - 1 - x$. NONE

10. A power series representation for $f(x) = \sin^2 x + \cos^2 x$ is

A. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ B. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ C. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+2n+1)}}{(2n)!(2n+1)!}$ D. 1

11. The sum of $\sum_{n=1}^{\infty} nx^{n-1}$, where -1<x<1, is

A. $n\frac{1}{1-x}$ B. $-\frac{1}{(1-x)^2}$ C. $\left(\frac{1}{1-x}\right)'$ D. $x+1$

12. The Maclaurin series representation for $\ln(1+2x)$ is

A. $\sum_{n=1}^{\infty} \frac{x^n}{n}$ B. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$ C. $\sum_{n=1}^{\infty} \frac{(-2)^{n-1} x^n}{n}$ D. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} (2x)^n}{n}$

13. The Lagrange remainder if the Maclaurin polynomial of degree 5 is used for $e^{0.3}$ is

A. $\sum_{n=0}^{5} \frac{0.3^n}{n!}$. B. $\frac{e^X 0.3^5}{5!}$ for some X, $0 \le X \le 0.3$.

C. $\frac{e^X 0.3^6}{6!}$ for some X, $0 < X < 0.3$.

14. Write down the binomial series representation for $\sqrt{\frac{1}{1+x}}$. Do **nor** simplify.

15. The following sequence of functions converges uniformly to 0:

A. $\{x^n\}$, $0 \le x \le 1$. B. $\{x^n\}$, $\frac{1}{2} \le x \le 1$. C. $\{x^n\}$, $0 \le x < 1$. D. $\{x^n\}$, $0 < x \le 1$.

16. Which of the following statements are true.

A. If $\{f_n(x)\}$ converges pointwise to $f(x)$ then it converges uniformly to $f(x)$.

B. If $\{f_n(x)\}$ converges uniformly to $f(x)$ then it converges pointwise to $f(x)$.

C. If $\{f_n(x)\}$ converges uniformly to $f(x)$ then $\{f_n'(x)\}$ converges pointwise to $f'(x)$.

D. If $\{f_n(x)\}$ converges uniformly to $f(x)$ and all $f_n(x)$ are continuous, then so is $f(x)$.