2,3,4.

136.151: Test #4
20 minutes

Name:____________________ **Student Number:** _______

1. Is the graph of the function $y = \frac{1}{x+1}$, $x \geq 0$ concave up or concave down? Justify your answer.

Solution. Compute $y' = \frac{-1}{(x+1)^2}$ and $y'' = \frac{1}{(x+1)^3}$. Since $\frac{1}{(x+1)^3}$ is positive for $x \geq 0$, it follows that the graph of our function is concave up over its domain.

2. Find and classify the absolute extrema of the function $f(x) = x^2 - 4x + 3$ over the interval $[0,3]$.

Solution. Compute $f'(x) = 2x - 4$. Since x=2 is the only solution of $2x - 4 = 0$, this is the only critical point (and it is in the given interval). We compute $f(2) = -1$ (at the critical point) and $f(0) = 3$, $f(3) = 0$ (at the edges of the interval). It follows that –1 is the absolute minimum of the function (and it happens for x=2), while 3 is the absolute maximum of the function (and it happens at x=0).

3. The side of a cubic box is increasing at the rate of 3 m/sec. How fast is the volume of the box increasing at the moment when the side of the box is 10 m.?

Solution. Denote the side of the cube by *x*. Then the volume of the box is $V = x^3$. We differentiate with respect to the time *t* to get (by the chain rule) that $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$. At the given moment (when *x*=10) we have $\frac{dV}{dt} = 3(100)(3) = 900m^3/\sec$.