2,3,4.

136.151: Test #2 Solutions

1. Evaluate the limit or show it does not exist. In the latter case check if the limit is $+\infty$, $-\infty$ or neither. Identify any horizontal or vertical asymptotes from the two limits below (Do not compute other limits; just extract the information regarding asymptotes from the two limits below). **(a)** $\lim_{x\to 1^-}\frac{x+1}{x-1}$ **(b)** $\lim_{x\to\infty}\frac{\sqrt{x}+1}{\sqrt{x}-1}$

(a) $\lim_{x\to 1^-}\frac{x+1}{x-1}=-\infty$ Argument: as x approaches 1 from the left hand side, x-1 approaches 0 through negative numbers while at the same time x+1 approaches 2. It follows that the quotient $x+1/x-1$ is close to 2 divided by a negative number that is close to 0. That yields a very small negative number that tends to $-\infty$ as x tends to 1 from the left.

(b)

$$\lim_{x\to\infty}\frac{\sqrt{x}+1}{\sqrt{x}-1}=\lim_{x\to\infty}\frac{\sqrt{x}(1+\frac{1}{\sqrt{x}})}{\sqrt{x}(1-\frac{1}{\sqrt{x}})}=\lim_{x\to\infty}\frac{(1+\frac{1}{\sqrt{x}})}{(1-\frac{1}{\sqrt{x}})}=1$$

The first limit tells us that x=1 is a vertical asymptote for the function $\frac{x+1}{x-1}$, while the second limit tells us that y=1 is a horizontal asymptote for the function $\frac{\sqrt{x}+1}{\sqrt{x}-1}$.

2. Compute $f'(-1)$ if $f(x)=x^2-1$ **using ONLY the definition of the derivative** of a function. Does the function $f(x)$ increase or does it decrease at the moment when $x=-1$?

$$f'(-1)=\lim_{h\to 0}\frac{f(-1+h)-f(-1)}{h}=\lim_{h\to 0}\frac{[(-1+h)^2-1]-[(-1)^2-1]}{h}=\lim_{h\to 0}\frac{-2h+h^2}{h}=\lim_{h\to 0}(-2+h)=-2$$

Since the derivative is negative, the function decreases at that moment.

3.

(a) Compute $f'(1)$ if $f(x)=\sqrt{x}+3$. You may use any of the properties or theorems from the sections covered in this test.

(b) Compute $g'\left(\frac{1}{2}\right)$ if $g(x)=\frac{x^3-7x}{2x}$. You may use any of the properties or theorems from the sections covered in this test.

(c) Use (a) and (b) to show that the tangent line to the curve $f(x)=\sqrt{x}+3$ at the point when $x=1$ is parallel to the tangent line to the curve $g(x)=\frac{x^3-7x}{2x}$ when $x=1/2$.

(a) $f'(x)=(x^{1/2}+2)'=\frac{1}{2}x^{-1/2}$ and so $f'(1)=\frac{1}{2}$.

(b) Notice first that $g(x)=\frac{1}{2}x^2-\frac{7}{2}$. So, $g'(x)=\frac{1}{2}2x=x$ and so $g'(1/2)=1/2$.

(c) Parts (a) and (b) told us that the slopes of the tangent lines of the two functions at the two points are equal (to 1/2). Consequently these tangent lines are parallel.