

1.

**136.151: Test #1**  
**20 minutes**

Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

1. Consider the following three lines.

$$l_1 : 2y = 2x + 1$$

$$l_2 : y - x = 10$$

$$l_3 : y = -x + 4$$

Which of these lines are mutually parallel, which are mutually perpendicular? Why?

**Solution.**

For  $l_1$  we find that  $y = x + \frac{1}{2}$  so that the slope of that line is  $m_1 = 1$ .

For  $l_2$  we find that  $y = x + 10$  so that the slope of that line is  $m_2 = 1$ .

For  $l_3$  we are given that  $y = -x + 4$  so that the slope of that line is  $m_3 = -1$ .

Since  $m_1 = m_2$ , the first two lines are parallel. Since  $m_1 = -\frac{1}{m_3}$ , the first and the third line are perpendicular. Consequently, so are the second and the third.

2. Find the radius and the center of the circle defined by  $x^2 + y^2 - 2y = 0$ . Show your work.

**Solution.**

Since  $x^2 + y^2 - 2y = x^2 + (y-1)^2 - 1$ , the equation becomes  $x^2 + (y-1)^2 - 1 = 0$ , or equivalently  $x^2 + (y-1)^2 = 1$ . From there we find that the center of the circle is  $(0,1)$  and its radius is 1.

3. (a) Show that  $f(x) = x^2 + 4$  is an even function.

(b) Which of the functions  $g(x) = x^3 + 1$  and  $h(x) = x^3 + x$  is odd? Is any of these two functions even?

**Solution.**

(a)  $f(-x) = (-x)^2 + 4 = x^2 + 4 = f(x)$  so the function is indeed even.

(b)  $h(x)$  is odd. Both are not even.

4. Compute  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{(x+3)(x-1)}$ . Show your work.

**Solution.**

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{(x+3)(x-1)} = \lim_{x \rightarrow 1} \frac{(x-2)(x-1)}{(x+3)(x-1)} = \lim_{x \rightarrow 1} \frac{(x-2)}{(x+3)} = \frac{-1}{4}.$$