Values

[10] 3. Is the function $f(x)=|x-2|+x$ continuous at $x=2$? Is it differentiable at $x=2$? Justify your answer using the definition of continuity and the right and left derivatives.

CONTINUITY: $f(2)=2$

$$\lim_{x\to 2} f(x)=\lim_{x\to 2}|x-2|+2=2$$

SINCE $\lim_{x\to 2} f(x)=f(2)$ THE FUNCTION IS CONTINUOUS AT $x=2$.

DIFFERENTIABILITY:

$$f'_-(2)=\lim_{h\to 0^-}\frac{f(2+h)-f(2)}{h}=$$

$$=\lim_{h\to 0^-}\frac{|2+h-2|+(2+h)-[|2-2|+2]}{h}=$$

$$=\lim_{h\to 0^-}\frac{|h|+h}{h}=\lim_{h\to 0^-}\frac{-h+h}{h}=0$$

$$f'_+(2)=\lim_{h\to 0^+}\frac{f(2+h)-f(2)}{h}=$$

$$=\lim_{h\to 0^+}\frac{|2+h-2|+(2+h)-[|2-2|+2]}{h}=$$

$$=\lim_{h\to 0^+}\frac{|h|+h}{h}=\lim_{h\to 0^+}\frac{h+h}{h}=2$$

SINCE ~~THESE~~ $f'_-(2)$ & $f'_+(2)$ ARE NOT EQUAL THE FUNCTION IS NOT DIFFERENTIABLE AT $X=2$.