Values

[6] 1. Evaluate the limit (or explain why it does not exist).

(a) $\lim_{t \to 1} \frac{t-1}{\sqrt{t^2+t}-\sqrt{2t}} =$

$$= \lim_{t \to 1} \frac{t-1}{\sqrt{t^2+t}-\sqrt{2t}} \cdot \frac{\sqrt{t^2+t}+\sqrt{2t}}{\sqrt{t^2+t}+\sqrt{2t}} =$$

$$= \lim_{t \to 1} \frac{(t-1)(\sqrt{t^2+t}+\sqrt{2t})}{t(t-1)} =$$

$$= \lim_{t \to 1} \frac{\sqrt{t^2+t}+\sqrt{2t}}{t} = \frac{\sqrt{2}+\sqrt{2}}{1} = 2\sqrt{2}$$

[6] (b) $\lim_{x \to 2^-} \frac{|x-2|}{x^2-2x} =$

$$= \lim_{x \to 2^-} \frac{-(x-2)}{x(x-2)} = \lim_{x \to 2^-} -\frac{1}{x} = -\frac{1}{2}$$

[6] (c) $\lim_{x \to \infty} \sqrt{x^2+1000}-x =$

$$= \lim_{x \to \infty} \frac{(\sqrt{x^2+1000}-x)(\sqrt{x^2+1000}+x)}{\sqrt{x^2+1000}+x} =$$

$$= \lim_{x \to \infty} \frac{1000}{\sqrt{x^2+1000}+x} = 0$$

Values

2. Find the derivative of the function. Do not simplify your answer.

[6] (a) $f(x)=\dfrac{(3x+1)^{1/3}}{1+\sqrt{2x-1}}$

$$f'(x)=\frac{\left[(3x+1)^{1/3}\right]'\cdot(1+\sqrt{2x-1})-(3x+1)^{1/3}\cdot(1+\sqrt{2x-1})'}{(1+\sqrt{2x-1})^2}=$$

$$=\frac{\frac{1}{3}(3x+1)^{-2/3}\cdot 3(1+\sqrt{2x-1})-(3x+1)^{1/3}\cdot\frac{1}{2\sqrt{2x-1}}\cdot 2}{(1+\sqrt{2x-1})^2}$$

[6] (b) $y=\left(\pi+\dfrac{1}{x}\right)^{10}\left(x^2-10\right)^{50}$

$$y'=\left[\left(\pi+\frac{1}{x}\right)^{10}\right]'(x^2-10)^{50}+\left(\pi+\frac{1}{x}\right)^{10}\cdot\left[(x^2-10)^{50}\right]'=$$

$$=10\left(\pi+\frac{1}{x}\right)^{9}\cdot\left(-\frac{1}{x^2}\right)(x^2-10)^{50}+\left(\pi+\frac{1}{x}\right)^{10}\cdot 50(x^2-10)^{49}\cdot 2x$$

Values

[10] 3. Is the function $f(x)=|x-2|+x$ continuous at $x=2$? Is it differentiable at $x=2$? Justify your answer using the definition of continuity and the right and left derivatives.

CONTINUITY: $f(2)=2$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} |x-2|+2 = 2$$

SINCE $\lim_{x\to 2} f(x) = f(2)$ THE FUNCTION IS CONTINUOUS AT $X=2$.

DIFFERENTIABILITY:

$$f'_-(2) = \lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h} =$$

$$= \lim_{h\to 0^-} \frac{|2+h-2| + (2+h) - [|2-2|+2]}{h} =$$

$$= \lim_{h\to 0^-} \frac{|h|+h}{h} = \lim_{h\to 0^-} \frac{-h+h}{h} = 0$$

$$f'_+(2) = \lim_{h\to 0^+} \frac{f(2+h)-f(2)}{h} =$$

$$= \lim_{h\to 0^+} \frac{|2+h-2| + (2+h) - [|2-2|+2]}{h} =$$

$$= \lim_{h\to 0^+} \frac{|h|+h}{h} = \lim_{h\to 0^+} \frac{h+h}{h} = 2$$

SINCE ~~THESE~~ $f'_-(2)$ & $f'_+(2)$ ARE NOT EQUAL THE FUNCTION IS <u>NOT</u> DIFFERENTIABLE AT $X=2$.

Values

[10] 4. Find the equation of the tangent line to the curve $2x^3 - x^2y^2 + 3y - 4 = 0$ at the point $(1,2)$. What is the equation of the normal line to the curve at that point? (Recall that the normal line to a curve is the line perpendicular to the tangent line.)

$$(2x^3 - x^2y^2 + 3y - 4)' = 0' \quad \text{(WITH RESPECT TO X)}$$

$$6x^2 - (2xy^2 + x^2 2yy') + 3y' = 0$$

$$y' = \frac{2xy^2 - 6x^2}{-2x^2y + 3}.$$

AT $(1,2)$ WE HAVE $y' = -2$.

TANGENT LINE: $y = -2x + b$ AND

SINCE $2 = -2 + b$ WE FIND THAT $b = 4$

SO $\rightarrow$ $\underline{y = -2x + 4}$

SLOPE OF NORMAL LINE IS $-\frac{1}{-2} = \frac{1}{2}$.

NORMAL LINE:

$\underline{y = \frac{1}{2}x + 3/2}$

Values

[10] 5. A ball is thrown upward from ground level with an initial speed of 9.8 m/s so that its height in metres after t seconds is given by

$$y = 9.8t - 4.9t^2.$$

(a) What is the acceleration of the ball at any time ?

VELOCITY : $v(t) = y' = 9.8 - 9.8t$ m/s.

ACCELERATION: $a(t) = y'' = -9.8$ m/s^2.

(b) How high can the ball go?

AT THE HIGHEST POINT $v(t) = 0$. SOLVE $9.8 - 9.8t = 0$, GET $t = 1$. AT THAT MOMENT $y\big|_{t=1} = 4.9$ m.

(c) How fast is it moving when it strikes the ground?

WHEN THE BALL STRIKES THE GROUND $y = 0$. SOLVE $9.8t - 4.9t^2 = 0$ TO GET $t = 0$ (start) AND $t = 2$ (finish.). SO THE BALL STRIKES THE GROUND AFTER 2 SECONDS.

VELOCITY AT THAT MOMENT: $v(2) = 9.8 - 9.8(2) =$

$= -9.8$ m/s.

SPEED $= |v(2)| = 9.8$ m/s

Values

[3] 6. [Bonus question]. Suppose that $f(x)$ is a function that satisfies the equation

$$f(x+y) = f(x) + f(y) + x^2y + xy^2$$

for all real numbers x and y. Suppose also that $\lim_{x\to 0} \frac{f(x)}{x} = 1$.

(a) Find $f(0)$. (Hint: Let $x = y = 0$ in the equation.)

$$f(0+0) = f(0) + f(0) + 0 + 0$$

so $f(0) = 2f(0)$, so $f(0) = 0$

(b) Find $f'(0)$. (Hint: Use the definition of derivative.)

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} =$$

$$= \lim_{h\to 0} \frac{f(h)}{h} = 1 \quad \text{(GIVEN !)}$$

(c) Find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$$

$$= \lim_{h\to 0} \frac{\overset{=0}{f(x)} + f(h) + x^2h + xh^2 - \overset{=0}{f(x)}}{h} =$$

$$= \lim_{h\to 0} \frac{f(h) + x^2h + xh^2}{h} = \lim_{h\to 0}\left(\frac{f(h)}{h} + x^2 + xh\right) =$$

$$= 1 + x^2 \;//$$