B16.

136.130: Test #4
20 minutes
Solutions

1. Suppose $\mathbf{u} = (1,0,-2)$, $\mathbf{v} = (1,1,-1)$, $\mathbf{w} = (0,1,-2)$. Compute if possible, otherwise state it is not possible.

(a) $\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w})$
(b) $(\mathbf{u} \bullet \mathbf{v}) \times \mathbf{w}$
(c) $\mathbf{u} \times (\mathbf{v} \times (0,0,0))$

Solution. (a) $\mathbf{u} \bullet (\mathbf{v} \times \mathbf{w}) = \mathbf{u} \bullet \left(\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}, -\begin{vmatrix} 1 & -1 \\ 0 & -2 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \right) = \mathbf{u} \bullet (-1,2,1) = -1 - 2 = -3$

(b) $(\mathbf{u} \bullet \mathbf{v}) \times \mathbf{w}$ does not exist
(c) $\mathbf{u} \times (\mathbf{v} \times (0,0,0)) = (0,0,0)$

2. Find the equations of the line through the point $A(3,4,5)$ that is parallel to the line $x = 2 - t,\ y = -t,\ z = 1 + 2t$.
Solution. Since $(-1,-1,2)$ is parallel to the given line, it is also parallel to the line we want. So, the equations of the latter are $x = 3 - t,\ y = 4 - t,\ z = 5 + 2t$.

3. Find the equation of the plane that passes trough the point $A(3,4,5)$ and is perpendicular to the line $x = 2,\ y = 2t,\ z = 2 + 2t$.

Solution. Since $(0,2,2)$ is perpendicular to the plane, it is parallel to the line we want. So, the equations of the line are $x = 3,\ y = 4 + 2t,\ z = 5 + 2t$.

4. Find the point of intersection of the line $x = 2,\ y = 2t,\ z = 2 + 2t$ and the plane $x + y + z = 4$.
Solution. Substituting $x = 2,\ y = 2t,\ z = 2 + 2t$ in the equation of the plane gives $x + 2t + 2 + 2t = 4$ which tells us that $t = 0$. So, the point of intersection is $(2,0,0)$