B15.

136.130: Test #3
Solutions

1. Solve the following system using Cramer's rule. No points will be awarded if other methods are used.

$$\begin{aligned} 2x &+ y &= 5 \\ x &- y &= 1 \end{aligned}$$

Solution. With $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$, $A_1 = \begin{bmatrix} 5 & 1 \\ 1 & -1 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}$, we have $x = \dfrac{\det(A_1)}{\det(A)} = \dfrac{-6}{-3} = 2$ and $y = \dfrac{\det(A_2)}{\det(A)} = \dfrac{-3}{-3} = 1$.

2. Consider the following matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{bmatrix}$. Compute the cofactor C_{23}. Show your work.

Solution. $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{vmatrix} = -4$

3. Suppose $\mathbf{u} = (1,2,0)$, $\mathbf{v} = (0,0,-2)$.

(a) Compute $(2\mathbf{u} + \mathbf{v}) \bullet \mathbf{u}$

(b) Are **u** and **v** perpendicular. Justify your answer.

Solution.

(a) $(2\mathbf{u} + \mathbf{v}) \bullet \mathbf{u} = ((2,4,0) + (0,0,-2)) \bullet (1,2,0) = (2,4,-2) \bullet (1,2,0) = 10$

(b) Since $\mathbf{u} \bullet \mathbf{v} = 0$ the two vectors are perpendicular.